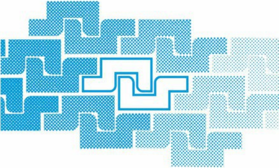

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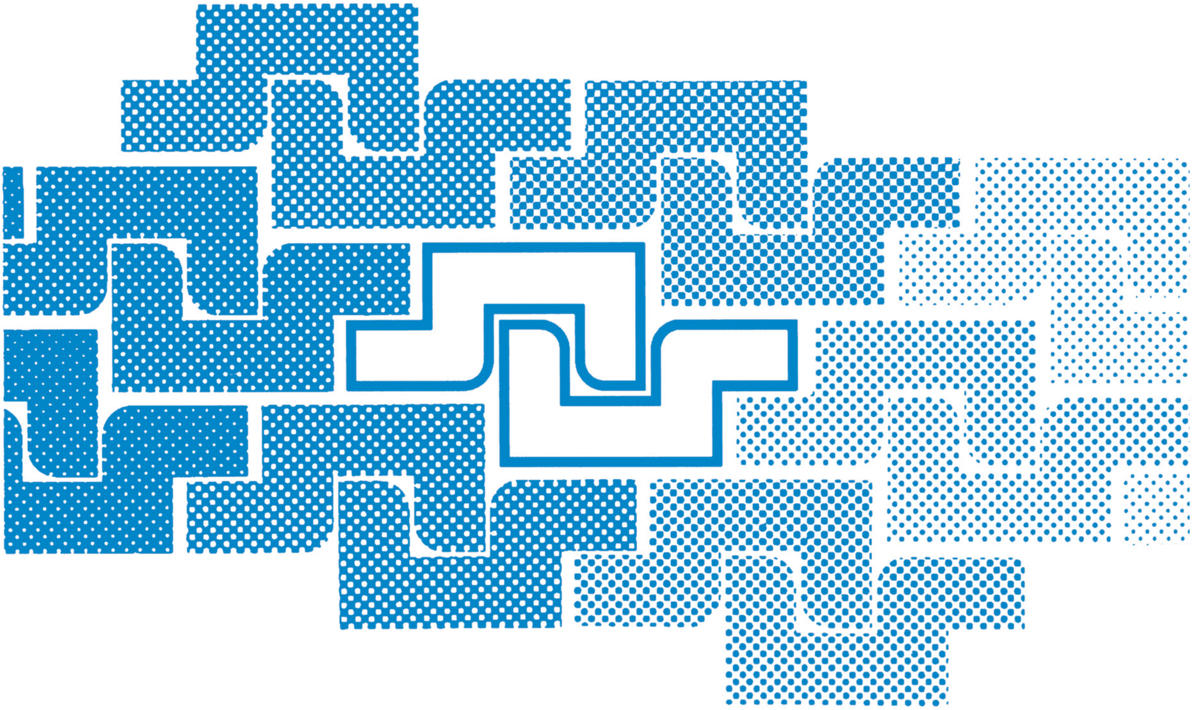
NATO Science Series

II. Mathematics, Physics and Chemistry – Vol. 187



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Series II: Mathematics, Physics and Chemistry – Vol. 187

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Contents

Preface

The field of cosmology is currently undergoing a revolution driven by dramatic observational progress and by novel theoretical scenarios imported from particle physics. In particular, two most remarkable results were recently obtained from measurements of the angular spectrum of the fluctuations in the Cosmic Microwave Background (CMB) radiation providing convincing evidence that the Universe is nearly flat and from the Hubble diagram of distant supernovae indicating an accelerating expansion rate, which implies the existence of some dark energy as the dominant component of the Universe. Indeed, the next decade will benefit from high quality data on cosmology from different major experiments and observatories, with a particular important contribution from space missions such as WMAP, Planck Surveyor, XMM and SNAP among others. On one side, cosmologists believe they understand the origin of the main ingredients which allow a coherent description of the Universe from its very early phase, namely inflation, to the actual epoch which accounts for the origin of the primordial fluctuations, allowing predictions of their imprints in the cosmic microwave sky and leading to the large scale structure of the Universe as observed. On the other side, the existence of a non-zero vacuum density is certainly one of the most astonishing results of modern fundamental physics. Understanding its nature and its origin will be one of the major directions of research in the following years. In view of the intensive current activity in the field, a School fully dedicated to these both sides in cosmology was timely. This 11-days NATO Advanced Study Institute took place in the lovely setting of the *Institut d'Études Scientifiques de Cargèse (Corse, France)* and was attended by about 80 participants from several countries. These proceedings contain the papers that were presented during the School and which covered the following fields : quintessence/dark energy; inflation; CMB: anisotropies and polarization; large scale structure; clusters of galaxies; gravitational lensing; galaxy formation; dark matter; supernovae and the accelerating expansion of the Universe.

ALAIN BLANCHARD & MONIQUE SIGNORE

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The success of the highly productive, friendly and relaxed School **Frontiers of the Universe** is largely due to the local and flawless organization of the *Institut d'Études Scientifiques de Cargèse* and to the support provided by Dr Elisabeth Dubois-Violette, director of the Institut. This school will not have been possible without the financial support from NATO. Finally, we also gratefully acknowledge the financial support of CNRS, INSU, IN2P3, PNC, CNES and Collectivité Territoriale Corse.

Chapter 1

BASICS OF COSMOLOGY

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Introduction

We will begin by briefly reviewing the *General Cosmological Framework* in which the following lectures will fit the *Hot Big Bang*!
In practice, on the basis of three observational facts :
i) the Universe is currently in a state of uniform expansion ,
ii) the Universe is filled with photons that come from background blackbody radiation at a temperature of about 2.74 K ,
iii) the Universe is isotropic on large scales i.e. beyond nearly 1000 Mpc,
one can construct, from General Relativity, a generic cosmological model known as the *Hot Big Bang* or the *Standard Big Bang Model*.
This model, which ancestor is the Lemaî tre *Primeval atom*, is but a mere 40 years old and provides a description of some other observations like, in particular, the abundances of light elements.

The paper is organized as follows : in section 2, we briefly review the geometry and dynamics of the Universe and then give the Einstein-Friedman-Lemaître (hereafter EFL) equations; section 3 introduces some important quantities needed for observations; in section 4, we rapidly present some solutions of the EFL equations, i.e. some cosmological models; in section 5, the *Standard Big Bang Nucleosynthesis Model* is described while section 6 shows a statement of observations of primordial abundances; in section 7, we confront the predictions of the Standard Big Bang Nucleosynthesis (hereafter SBBN) model to the observations of the primordial abundances; a brief conclusion is

A. Blanchard and M. Signore (eds.), Frontiers of Cosmology, 1–22.

given in section 8.

Let us also mention the recent review "Precision Cosmology" - and references therein - due to Melchiorri et al. (2003) which develops many topics that are rapidly presented here.

1. Geometry and Dynamics

Geometry of 4-dimensional space-time

The fundamental idea of geometrical theory of gravity starts from the fact that we can assign four coordinates to any event observed in our vicinity, for instance in Cartesian coordinates (x, y, z, t). Locally, space appears flat. However this does not prejudge of the global shape of space : local observations put us in the same situation that lead people to think the earth was flat. Let us take the line element of a homogeneous 3D space which can be shown to be :

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{1 - k\left(\frac{r}{R}\right)^2}$$

where k is $-1, 0, 1$ accordingly to whether space is hyperbolic, flat or spherical. R is a characteristic size (in the spherical case, that is the radius of the 3D-sphere embedded in a 4D space).

We can add the time as the fourth coordinate, to build the equivalent of the Minkowski space-time element. We then get the Robertson-Walker line element after the change of variables $\frac{r}{R} \rightarrow r$:

$$ds^2 = -c^2dt^2 + R(t)^2[r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{1 - kr^2}] \tag{1.1}$$

Topology

The above line element depends on the local shape of space: the curvature (i.e. the value of k) is only a local property of space, its geometry, but does not tell us the *global* shape of space. For instance, the Euclidean plane is an infinite flat surface while the surface of a cylinder is a 2D-space which is flat everywhere but is finite in one direction. Identically, we may in principle derived the local geometry of space through General Relativity. It does not prejudge of the global topology of space. Only direct observations would allow to test what the topology actually is. Of course this will not be possible on scales larger than what can be observed (the horizon). We can therefore hope to prove that the Universe is finite, if it is small enough, but we could never know whether we are in a finite Universe of which the scale is larger than the horizon, or whether we are in an infinite Universe.

Dynamics

The function $R(t)$ which appears in the RW line element, is totally independent of any geometrical consideration. It can be specified only within a theory of gravity. Although General Relativity (GR hereafter) is at the starting point of modern cosmology, it is often of little use in practice as in most cases we are in the weak field regime, for which Newton theory is sufficient. Therefore, this lecture will say almost nothing about GR. The basic equation of GR relates the geometrical tensor G_{ij} to the energy-momentum tensor T_{ij}

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = 8\pi G T_{ij} \tag{1.2}$$

There exists a coordinates system, called the comoving coordinates, in which the matter is at rest, and the tensor T_{ij} is diagonal with $T_{00} = \rho$ and $T_{11} = T_{22} = T_{33} = p$, ρ being the density and p the pressure. A fundamental aspect of GR is that the source of gravity includes explicitly a term coming from the pressure : $\rho + 3p/c^2$. Finally, there is an analog of the Gauss theorem, that is the Birkhoff's theorem: if the matter distribution is spherical then the evolution of the radius of a given shell of matter does depend only on its internal content.

From the above rules, we can easily derive the equation for R(t). Let us consider a spherical region of radius a in a homogeneous distribution of matter. The equivalent Newtonian acceleration is:

$$\frac{d^2a}{dt^2} = g$$

with the acceleration being generated by the "mass" of the above spherical region $M(a)$:

$$g = -\frac{GM(a)}{a^2} = -\frac{4}{3}\pi G(\rho + 3p/c^2)a$$

The density term includes the effect of kinetic energy ($E = mc^2$!), so that energy conservation can be written inside the volume of the sphere, and elementary thermodynamics gives:

$$d(E_t) = d(\rho V c^2) = -pdV$$

leading to :

$$\dot{\rho} = -3(\frac{p}{c^2} + \rho)\frac{\dot{a}}{a}$$

From these two equations, the pressure can be eliminated, and, after having multiply both term by $\dot{a}$, the differential equation can be easily integrated. This leads to the following equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{Kc^2}{a^2}$$