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Limit States of Materials and Structures

Direct Methods

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Preface

To predict loading limits for structures and structural elements is one of the oldest and most important tasks of engineers. Among the theoretical and numerical methods available for this purpose, so-called “Direct Methods”, embracing Limit- and Shakedown Analysis, play an eminent role due to the fact that they allow rapid access to the requested information in mathematically constructive manners.

The collection of papers in this book is the outcome of a workshop held at Aachen University of Technology in November 2007. The individual contributions stem in particular from the areas of new numerical developments rendering the methods more attractive for industrial design, extensions of the general methodology to new horizons of application, probabilistic approaches and concrete technological applications.

The papers are arranged according to the order of the presentations in the workshop and give an excellent insight into state-of-the-art developments in this broad and growing field of research.

The editors warmly thank all the scientists, who have contributed by their outstanding papers to the quality of this edition. Special thanks go to Jaan Simon for his great help in putting together the manuscript to its final shape.
We hope you may enjoy reading it!

Aachen and Leicester,
September 2008

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The Linear Matching Method for Limit Loads, Shakedown Limits and Ratchet Limits

A.R.S. Ponter

Abstract The paper describes the application of the Linear Matching Method to the direct evaluation of limits associated with an elastic-perfectly plastic body subjected to cyclic loading. Methods for limit load and shakedown limit are followed by ratchet limits. The method is distinguished from other programming methods by ensuring that equilibrium and compatibility are satisfied at each stage. The method has been extended beyond the range of most other direct methods by including ratchet limits and high temperature material behaviour. Implementation is possible within the user routines of commercial finite element codes. The paper emphasise the theoretical characteristics of the method and discusses significant aspects of convergence, both theoretical and numerical. The application of the method to industrial Life Assessment problems and to geotechnical problems is summarized.

1 Introduction

Classically, numerical methods for Direct Method have relied upon the application of mathematical programming methods to limit load and shakedown limit theorems of plasticity and this is strongly reflected in other papers in this volume. Either upper or lower bound methods are possible depending on whether the approximating continuum descriptions correspond to equilibrium stress fields or compatible strain fields. The objective function, a load parameter, is then either maximized or minimized according the upper and lower bounds of classical plasticity.

This approach has both advantages and disadvantages. Mathematical programming procedures have progressed significantly in recent years and highly

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efficient solution methods are widely available producing fast and reliable solution methods. However, the general methodology relies upon the classical theorems of plasticity and extensions do not exist to behaviour outside shakedown or to other material behaviour, time dependent creep behaviour in particular. For this reason, alternative approaches have been considered that provide a more flexible approach to the formulation of direct methods. A number of methods discussed within the design community [6, 13, 21] have proved to have a common theme, the representation of stress and strain fields through linear problems with spatially varying moduli. This first occurred through Marriot's [21] observation that very good lower bound limit loads could be found by systematically decreasing the Young's modulus in regions of high stress in a standard linear elastic solution. This has the effect of reducing the maximum stress and hence increasing the load at which all stresses lie within yield. This simple procedure can be very successful in computing safe lower bounds but, to the present time, has not been developed into a method for evaluation the maximum lower bound, i.e. the limit load.

The Linear Matching Method [8, 23, 25, 26, 28] adopts the basic assumption that limit state solutions may be developed from linear solutions with spatially varying moduli and builds it into a programming method. In this paper, the method is described for an elastic-perfectly plastic body for the evaluation of all the possible limits; limit load, shakedown limit and ratchet limit. It is shown for shakedown that the method becomes a convergent upper bound method. Each iteration provides both a kinematically admissible strain rate history and an equilibrium distribution of residual stress (in the Galerkin sense), both upper and lower bounds are generated that become equal to the minimum upper bound at convergence. This lower bound generally does not monotonically increase. A dual method, based upon equilibrium stresses and the lower bound theorem also exists but appears not to converge for perfect plasticity [23].

The extension of the method to the ratchet limit is then discussed, based upon a general minimum theorem for perfect plasticity [14, 25]. This is followed by a summary of various applications to industrial problems.

2 Definition of the Problem

2.1 External Loads

We consider the problem of a body with volume V subjected to a cyclic history of load $\lambda P_i(x_i, t)$ over part, namely S_T, of the surface S, and a cyclic history of temperature $\lambda\theta(x_i, t)$ within V. On the remainder of S, namely S_U, the displacement rate $\dot{u}_i = 0$. Both load and temperature history have the same cycle time Δt and, in the following, we are concerned with the behaviour of the body in a typical cycle $0 \leq t \leq \Delta t$ in a cyclic state. The (positive) load parameter λ allows us to consider a class of loading histories.

2.2 Material Behaviour

Consider such a body composed of a solid under conditions of small strains where the total strain is the sum of a linearly elastic and perfectly plastic component,

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^p_{ij}, \tag{1}$$

where the plastic strains are associated with a strictly convex yield condition $f(\sigma_{ij}) = 0$

$$\begin{aligned} \dot{\varepsilon}^p_{ij} &= 0, \quad f < 0, \\ \dot{\varepsilon}^p_{ij} &= \dot{\alpha}\frac{\partial f}{\partial \sigma_{ij}}, \quad f = 0. \end{aligned} \tag{2}$$

The elastic moduli are assumed to be independent of temperature.

2.3 Structure of the Asymptotic Cyclic Solution

For the problem defined above the stresses and strain rates will asymptote to a cyclic state, where

$$\sigma_{ij}(t) = \sigma_{ij}(t + \Delta t), \, \dot{\varepsilon}_{ij}(t) = \dot{\varepsilon}_{ij}(t + \Delta t). \tag{3}$$

In the following we are concerned with the properties of this asymptotic solution.

Consider a typical cycle, $0 \leq t \leq \Delta t$. The cyclic solution may be expressed in terms of three components, the elastic solution, a transient solution accumulated up to the beginning of the cycle and a residual solution that represents the remaining changes within the cycle. The linear elastic solution (i.e. $\dot{\varepsilon}^p_{ij} = 0$) is denoted by $\lambda\hat{\sigma}_{ij}$ and $\lambda\hat{\varepsilon}_{ij}$. The general form of the stress solution is given by

$$\sigma_{ij}(x_i, t) = \lambda\hat{\sigma}_{ij}(x_i, t) + \bar{\rho}_{ij}(x_i) + \rho^r_{ij}(x_i, t), \tag{4}$$

where $\bar{\rho}_{ij}$ denotes a constant residual stress field in equilibrium with zero surface traction on S_T and corresponds to the residual state of stress at the beginning and end of the cycle. The history ρ^r_{ij} is the change in the residual stress during the cycle and satisfies

$$\rho^r_{ij}(x_i, 0) = \rho^r_{ij}(x_i, \Delta t) = 0. \tag{5}$$

The total plastic strain is similarly subdivided into two parts

$$\varepsilon^p_{ij} = \varepsilon^{pT}_{ij}(x_i) + \varepsilon^{pr}_{ij}(x_i, t), \tag{6}$$

where ε_{ij}^{pT} denotes the accumulation of plastic strain at the beginning of the cycle and ε_{ij}^{pr} denotes the additional plastic strain during the cycle.

The relationship between the transient and residual quantities is given by

$$\varepsilon_{ij}^{T} = C_{ijkl}\bar{\rho}_{ij} + \varepsilon_{ij}^{pT}, \tag{7}$$

$$\varepsilon_{ij}^{r} = C_{ijkl}\rho_{ij}^{r} + \varepsilon_{ij}^{pr}, \tag{8}$$

where both ε_{ij}^{T} and ε_{ij}^{r} are compatible strain fields. C_{ijkl} denotes the linear elastic compliance tensor. The cyclic solution is always non-unique to the extent that the transient plastic strain ε_{ij}^{pT} may contain an arbitrary additional compatible component. Note that

$$\varepsilon_{ij}^{pr}(x_i, \Delta t) - \varepsilon_{ij}^{pr}(x_i, 0) = \Delta\varepsilon_{ij}^{pr} = \int_0^{\Delta t} \varepsilon_{ij}^{pr}\,\mathrm{d}t \tag{9}$$

the accumulated plastic strain during a cycle which, for consistency may be added to ε_{ij}^{T}. Hence as both ε_{ij}^{T} and $\varepsilon_{ij}^{T}+\Delta\varepsilon_{ij}^{pr}$ are compatible then $\Delta\varepsilon_{ij}^{pr}$ is also compatible with a displacement field Δu_i, the accumulation of displacement per cycle.

This argument emphasises the crucial role of the residual plastic strain rate history $\dot{\varepsilon}_{ij}^{pr}$ in defining the cyclic state. If $\dot{\varepsilon}_{ij}^{pr}$ were known for $0 \leq t \leq \Delta t$ then $\rho_{ij}^{r}(x_i, t)$ is uniquely defined by the solution of the initial strain rate problem defined by the time derivative of (8),

$$\dot{\varepsilon}_{ij}^{r} = C_{ijkl}\dot{\rho}_{ij}^{r} + \dot{\varepsilon}_{ij}^{pr} \tag{10}$$

and the initial condition (5). The cyclic stress history (4) is then known except for $\bar{\rho}_{ij}$. However the additional requirement of the compatibility of $\Delta\varepsilon_{ij}^{pr}$ is sufficient to define $\bar{\rho}_{ij}$ and hence the entire history of stress in the cycle is known and the final condition of (5) is always satisfied. It is natural, therefore, to concentrate on a class of inelastic strain rate histories that have the same properties as the solution $\dot{\varepsilon}_{ij}^{pr}$. We will refer to all strain rate histories as $\dot{\varepsilon}_{ij}^{c}$ that accumulate over a cycle to a compatible strain increment $\Delta\varepsilon_{ij}^{c}$ as kinematically admissible (KA).

3 Shakedown Theorems

In this section we consider the case of shakedown. The shakedown limit can be seen to be that range of load multiplier $\lambda \leq \lambda_s$ for which the changing residual stress ρ_{ij}^{r} is zero, where λ_s is the shakedown limit. In the following we state the shakedown theorems and then convert the upper bound theorem into a form naturally aligned to the subsequent discussion of the Linear Matching Method.