

Differential-Algebraic Equations Forum

DAE-F

Bernd Simeon

# Computational Flexible Multibody Dynamics

A Differential-Algebraic Approach

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# Differential-Algebraic Equations Forum

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Bernd Simeon

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To Shery.

توانا بُودِ هَرکه دانا بُودِ

*Adorn thy mind with knowledge, for  
knowledge maketh thy worth.*

Firdausi, Persian poet (940–1020 CE)

# Preface

Flexible multibody dynamics is a rapidly growing field with various applications in vehicle analysis, aerospace engineering, robotics, and biomechanics. Overall, the mathematical models in this field seem reasonably settled and, in computational terms, great strides have been made over the last two decades as sophisticated software packages are nowadays capable of simulating highly complex structures with rigid and deformable components.

Written from the perspective of a numerical analyst, this monograph provides comprehensive information on both the mathematical framework as well as the numerical methods for flexible multibody dynamics. Such a presentation of the subject appears to be needed and should benefit not only graduate students and scientists working in the field but also those interested in time-dependent partial differential equations and heterogeneous problems with multiple time scales. At the same time, a number of open issues at the frontiers of research are addressed by taking a differential-algebraic approach and extending it to the notion of transient saddle point problems.

The results in this book are the product of research work undertaken during the past fifteen years. Parts of the material were also covered in graduate courses offered at the Universität Karlsruhe, the Technische Universität München, and the Technische Universität Kaiserslautern. Though it is self-contained in many respects, the text nonetheless calls for some basic knowledge of discretization schemes for ordinary differential equations and of the finite element method. Simulation examples illustrate the mathematical models and the computational techniques.

The interplay of modeling and numerics is a key feature in flexible multibody dynamics and mainly determines the methodology chosen in this monograph. This is reflected by the organization into two main parts with four chapters each. While the first part develops a detailed mathematical framework for systems of rigid and deformable bodies, the second introduces discretization methods in space and time and concentrates in particular on the problem of different time scales.

Though this outline seems to indicate a purely mathematical treatment of the subject, this book is definitely not intended for the exclusive use of specialists in numerical analysis. On the contrary, it should promote the communication between the



engineer and the mathematician, thus leading to a truly interdisciplinary cooperation with mutual benefits.

The origin of this book goes back a long way. After I had completed the Ph.D., it was my advisor Peter Rentrop who first drew my attention to the field of flexible multibody dynamics. It is a pleasure to thank him for his continuing interest in the subject and for his support over the years. The next colleague and friend I would like to thank is Claus Führer, with whom I had various inspiring discussions and who carefully read an earlier version of the manuscript. Many other colleagues have also contributed by providing suggestions and valuable remarks on specific issues: Martin Arnold, Peter Betsch, Severiano Gonzalez Pinto, Ernst Hairer, Domingo Hernández Abreu, Laurent Jay, Christian Lubich, Panos Papadopoulos, Linda Petzold, Werner C. Rheinboldt, John Strain, and Barbara Wohlmuth. Moreover, I wish to sincerely thank all my master and Ph.D. students who worked in this or related fields for their collaboration, their effort, and their patience. Special thanks go to Klaus Dressler and his coworkers at the Fraunhofer ITWM in Kaiserslautern for their hints on practical aspects and for supplying the realistic example in Sect. 8.5. Last but not least, I express my gratitude to Doris Hemmer-Kolb and Kirsten Höffler for the final proofreading and to Mario Aigner from Springer Verlag for the efficient handling of the manuscript.

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**Part I**  
**Mathematical Models**

# Chapter 1

## A Point of Departure

What is a flexible multibody system? How can we derive an adequate mathematical model? And what are the major computational challenges that we are facing here? This introductory chapter gives some preliminary answers and, at the same time, illustrates the objectives pursued by this monograph.

A multibody system is defined to be a collection of bodies and interconnection elements. Joints constrain the relative motion of pairs of bodies while springs and dampers act as compliant elements. Furthermore, the bodies possess a certain mass and geometry whereas the interconnections are treated as massless. These basic modeling assumptions apply to a large class of mechanical and structural systems such as vehicles, robots, mechanisms, and air- and spacecrafts. Even the field of biomechanics makes extensive use of multibody models.

Variational principles dating back to Euler and Lagrange characterize the motion of a multibody system. More precisely, these principles provide a methodology, also called *formalism*, to generate equations of motion for arbitrary systems that comprise rigid bodies and interconnection elements. With these formalisms at the core of today's sophisticated simulation software, the automatic generation of models for highly complex structures has become a straightforward task. Depending on the choice of coordinates for the position and orientation of each individual body, such rigid multibody models form a system of ordinary differential equations (ODEs) or, if constraints are present, a system of differential-algebraic equations (DAEs).

In addition to rigid bodies, flexible multibody systems contain elastic components. These systems are used in connection with applications in which the rigid body assumption is no longer valid, a field that has increasingly been in demand in recent years due to a strong trend towards lightweight and high-precision mechanical systems. Accordingly, flexible multibody systems combine models and simulation methods both from rigid body mechanics and from structural analysis. Since elastic bodies are governed by partial differential equations (PDEs) and rigid bodies by ordinary differential or differential-algebraic equations, the mathematical model of a flexible multibody system is naturally heterogeneous. One could also speak of a coupled system of discrete (rigid bodies) and continuous (elastic bodies) components where the interface or coupling conditions deserve particular attention. An-