

ERNST ZERMELO

Collected Works Gesammelte Werke

VOLUME II BAND II

Calculus of Variations,
Applied Mathematics,
and Physics

Variationsrechnung,
Angewandte Mathematik
und Physik



HEIDELBERGER AKADEMIE
DER WISSENSCHAFTEN



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Zermelo around 1910

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VOLUME II
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Calculus of Variations, Applied Mathematics, and Physics
Variationsrechnung, Angewandte Mathematik und Physik

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Preface to the Zermelo edition

This is a complete edition of the published works of Ernst Zermelo which moreover includes selected correspondence and unpublished manuscripts. Zermelo is generally acknowledged for his pioneering work in axiomatic set theory and for introducing the axiom of choice as a basic principle of mathematics. In contrast, his work in applied mathematics and physics, despite its originality, is hardly recognized or has even been attributed to others. This edition of Zermelo's collected works provides a picture of the entire mathematician. It appears in two volumes. The first volume comprises Zermelo's published papers in set theory and the foundation of mathematics together with isolated papers of an algebraic, analytic, or number-theoretic character. The second volume is dedicated to Zermelo's work in the calculus of variations, mathematical physics, and fluid dynamics. Both volumes are supplemented by selected notes and manuscripts, mainly from Zermelo's *Nachlass*, which throw additional light on his papers, reflect his point of view, or are unpublished continuations of published work. To the best judgment of the editors, the selected notes and manuscripts fully and faithfully represent the essential unpublished writings of Zermelo concerning mathematics. Nevertheless, a possible edition of a third volume comprising further unpublished notes and letters from the *Nachlass* has *expressis verbis* been left open.

Both volumes contain some writings by other authors which include contributions actually written by Zermelo or which react to criticism Zermelo had made. Details are given in the prefaces to the respective volumes.

In order to provide access to a wider audience, the original papers are printed face to face with English translations. As both versions use the same layout, it is easy to go from the translation to the original version and vice versa. The layout itself tries to preserve the appearance of the original papers. For details we refer to the editorial information below.

Each paper or coherent group of papers is preceded by an introductory note which comments on contents, motivation, aims, and influence of the paper(s) concerned. Written by an expert in the field, it came to its final form in discussions with the editors.

Each volume contains a full bibliography of Zermelo together with a schematic *curriculum vitae* which will enable the reader to become acquainted with the personal circumstances from which a paper arose. In addition, Volume I starts with a more detailed biographical sketch of Zermelo's life and work.

Many of these features found their inspiration in the exemplary edition of Kurt Gödel's collected works by Solomon Feferman, John W. Dawson, Jr., and others.

The edition of Zermelo's collected works has a prehistory. Already as early as 1912, at the age of 41 and faced with a serious recurrence of his tuberculosis,

Zermelo conceived plans for an edition of his collected papers, but did not pursue them when his health improved. In 1949, under likewise deplorable personal circumstances, he tried again, this time approaching several publishers, among them Springer-Verlag. But the difficult situation in post-war Germany precluded such an enterprise. Immediately after Zermelo's death, in 1953, the historian of mathematics Helmuth Gericke and the philosopher Gottfried Martin, who had gotten to know Zermelo in the 1930s in Freiburg, started work on a two-volume edition, in 1956 gaining Paul Bernays as a third editor. Support was provided by the Kant-Gesellschaft. However, the plans were not realized; in 1962 work on the edition came to a definite end.

When in early 2004 new plans for an edition of Zermelo's collected works became more concrete, they found the enthusiastic support of Martin Peters of Springer-Verlag. In discussions with him it became clear very quickly that the edition should provide English translations and detailed comments. As Zermelo had been a member of the Heidelberger Akademie der Wissenschaften, the editors turned to the academy for financial support. The application found warm backing of Hans Günter Dosch, then Sekretar of the class for mathematics and the sciences of the academy. The application was successful. Even more, besides providing generous funding, the academy offered to let the edition appear in its regular series of publications of the class for mathematics and the sciences published by Springer-Verlag.

The editors wish to express deep gratitude to the Heidelberg academy for their ideal, financial support and to Springer-Verlag for their open-minded cooperation. In particular, many thanks go to Hans Günter Dosch and Martin Peters.

Freiburg, Toronto, and Boston
September 2009

Heinz-Dieter Ebbinghaus
Craig G. Fraser
Akihiro Kanamori

Preface to volume II

This second volume concludes the edition of Ernst Zermelo's collected works. The volume focuses on his contributions mainly to analysis and physics. Except for an excursion into physical chemistry (*Riesefeld and Zermelo 1909*), the papers come from the decade around 1900 when Zermelo was in Berlin and Göttingen and about two years around 1930 when he was in Freiburg. They are accompanied by three items found in Zermelo's and in David Hilbert's *Nachlass*. For orientation especially about the personal circumstances accompanying the genesis of the papers, the volume starts with Zermelo's *curriculum vitae*, the one given in volume I.

Zermelo's works of an applied character may hold pioneering ideas and insights, but they did not receive the attention they deserved. One reason may be the sheer diversity of topics he treated. Of course, one should also take into consideration that starting soon after the turn of the century his mathematical work shifted elsewhere for more than two decades, to set theory and mathematical logic, research in these disciplines leading him to his most influential scientific achievements.

The Berlin-Göttingen period comprises three topics: the calculus of variations, the kinetic theory of gases, and hydrodynamics.

The engagement with the calculus of variations started with Zermelo's Ph.D. thesis (*1894*), written at the University of Berlin under the guidance of Hermann Amandus Schwarz.

The engagement in the kinetic theory of gases started in 1896, also in Berlin, when Zermelo became an assistant to Max Planck. It lasted for about ten years. Its best-known part, a controversy with Ludwig Boltzmann, is described and analyzed here in full with the inclusion of *Boltzmann 1896, 1897*.

Zermelo's interest in meteorology led him to hydrodynamics, work that culminated in his 1899 *Habilitation* thesis (*1902a, s1902b, s1902c*) in Göttingen.

In the late 1920s, Zermelo came back to his "old, even though mostly unhappy love for the 'applications'". The starting paper (*1928*) on the evaluation of chess tournaments, with its early use of the maximum likelihood method, was to remain unknown until several decades later other people rediscovered his methods and results. Motivated by the circumnavigation of the earth by the airship Graf Zeppelin in August 1929, Zermelo wrote two papers (*1930c, 1931a*) on optimal steering methods of airships. Soon, however, this return to mathematics of an applied character came to an end when Zermelo got involved in a serious foundational debate which fully occupied what strength was left him after a serious illness.

The introductory notes are a crucial part of the Zermelo edition. Those who agreed to comment on a paper or a group of papers in this volume generously

shared their experience and knowledge with us and the potential reader. We at times had involved discussions toward securing the most informative and accurate presentations, and we appreciate the professionalism that was brought to bear.

The translations of the original papers were carried out by Enzo de Pellegrin. We again admire his extraordinary care and his feeling for both languages when handling Zermelo's style with its richness in nuances and its involved sentential structures. The introductory notes of Rüdiger Thiele were translated by David Kramer who with diligence and care successfully mirrored the style of the original German.

We express our gratitude to all who have supported us during our work. In this connection we would like to mention Ruth Allewelt from Springer-Verlag, Andrea Köhler and Petra Möws of Le-Tex Publishing Services, and Marlies Würth, the librarian of the Freiburg Mathematical Institute.

Again, Martin Peters of Springer-Verlag was ready to offer valuable help and advice.

We appreciate that Craig Fraser, while not being able to continue with his participation in the edition, was ready to contribute two substantial introductory notes.

Freiburg and Boston
December 2012

Heinz-Dieter Ebbinghaus
Akihiro Kanamori

Editorial information

Layout. The layout of the texts as well as of the translations mirrors the layout of the originals. Emphasized words, i.e., words in italics or words spaced out or consisting of small capitals, are given in italics. Original pagebreaks are indicated in the texts by “|”, and the number of the new original page beginning there is given on the margin.

Editorial annotations. These are set in double square brackets “[]”.

Misprints and errors. Small textual errors in the originals are tacitly corrected; larger ones are corrected with the corrections commented on in editorial annotations.

Wrong words or words missing in the originals have been replaced or added in double square brackets.

Misprints in mathematical expressions in the originals are not corrected in the texts. They are, however, corrected in the translations and noted by an editorial annotation.

References. In the texts Zermelo’s references to the literature are not altered. Translations as well as introductory notes refer to the main bibliography at the end of the volume instead and have the form *author(s) year of appearance*, followed by an additional index *a, b, c, . . .* if necessary. An example: *Hahn and Zermelo 1904*. If the authors are clear from the context, their names may be omitted; in such a case, *1904* may be short for *Hahn and Zermelo 1904*. References to page numbers are kept in both the texts and the translations; they can be traced via the original pagebreaks and the original page numbers provided in the texts.

Footnotes. Whereas the translations use natural numbers in ascending order as footnote marks, the texts preserve the original marks. It may thus happen that a page of the text may contain identical footnote marks. In such cases the original page numbers on the margin allow for quick correlation of mark and footnote.

Figures. Whenever possible, a figure is located at the same position as in the original. If this is not possible for a figure, say Fig. *n*, then its original position is indicated on the margin by “Fig. *n*” and the figure itself appears as close as possible, at worst on the top of the next page.

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