Differential-Algebraic Equations Forum

DAE-F

Achim Ilchmann
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Surveys in Differential-Algebraic Equations III

Springer

Differential-Algebraic Equations Forum

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Differential-Algebraic Equations Forum

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Surveys in Differential-Algebraic Equations III

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Preface of Surveys in Differential Algebraic Equations III

We are pleased to present the third volume of survey articles in various fields of differential-algebraic equations (DAEs), and we stress that a fourth volume will appear within the series “Differential-Algebraic Equations Forum”.

In this volume, we again extend the list of survey articles in the sense that they are of theoretical interest and equally relevant to applications.

The chapter “The Flexibility of DAE Formulations” shows that DAEs are not only the outcome of modeling; they may further lead to more elegant formulations in control and observer design problems, their numerical solution, and simulation. In the chapter “Reachability Analysis and Deterministic Global Optimization of DAE Models”, an overview on (optimal) control of parameterized DAEs is given. The chapter “Numerical Linear Algebra Methods for Differential-Algebraic Equations” is about numerical treatment of controller design and optimal control problems for large-scale differential-algebraic systems. The final chapter “Boundary-Value Problems for Differential-Algebraic Equations: A Survey” is a survey about boundary value problems for DAEs. Problems of this kind occur, for instance, in optimal control.

We hope that this issue will contribute to complete the picture of the latest developments in DAEs. The collection of survey articles may also indicate that DAEs are now an established field in applied mathematics.

Ilmenau, Germany — Achim Ilchmann
Hamburg, Germany — Timo Reis
May 2015

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The Flexibility of DAE Formulations

Stephen L. Campbell

Abstract There has been extensive research on DAEs and their applications. One major reason given for the usefulness of DAEs is that they are the initial way that many complex systems are most naturally modeled. But there are other ways that DAE formulations are useful. This survey focuses on a number of problems where the extra flexibility of a DAE formulation permits the solution of a problem that would be hard to solve otherwise.

Keywords Delays • Differential-algebraic equation • Numerical methods • Observer • Optimal control

MSC: 34A09, 65L80, 93B07, 49J15, 34A40

1 Introduction

There has been extensive research on differential algebraic equations (DAEs) and their applications. Note the books [1, 22, 33, 61–63, 84, 86] and such survey papers as [5, 24]. One major reason given for the usefulness of DAEs is that they are the initial way that many complex systems are most naturally modeled. This is especially true in chemical, electrical, and mechanical engineering and with models formed by interconnecting various submodels. But there are other ways that DAE formulations are useful. This survey focuses on a number of problems where the extra flexibility of a DAE formulation permits the solution of a problem that would be hard to solve otherwise. This survey takes the form of carefully chosen case studies where the DAE formulation has been found useful. Our examples are taken from work on control problems, their numerical solution, and simulation. Some examples are from our work and some is from the work of others. No attempt is

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made at completeness. So perhaps "essay" is a more accurate word to describe this paper than "survey."

We shall assume that the reader is familiar with what a DAE is. In particular, we will assume that readers have heard of the index of a DAE. There are several definitions of index. We take the differential index based off the derivative array as discussed just before (2.40). See also [22, 23, 62]. However, we do not assume that they are familiar with the different control and numerical topics we discuss. For material that has appeared in journal articles we omit some of the proofs and technical detail unless they are relevant to the point being made. For material that has only appeared in conference papers, especially if the proceedings are not immediately accessible, more details are provided. In sections with material that has not appeared anywhere full details in establishing the statements are given.

Section 2 will give some examples from control theory and in particular observer design. We will not discuss the design of observers for DAEs, this is done, for example, in [24]. Rather, we will present two different examples where the flexibility of a DAE formulation when designing observers can be exploited. In Sect. 2.1 the use of a DAE observer allows us to get linear error dynamics which is very useful in observer design. Section 2.2 discusses the estimation of disturbances. Section 3 turns to the examination of optimal control problems. It turns out that the advantages and disadvantages of a DAE formulation are highly dependent on the type of numerical methods used. We will focus on direct transcription both because it is widely used and because the computational theory is not always what one first thinks it is. Section 3 starts by describing what direct transcription is. Then two illustrations of the advantages of a DAE formulation are given in Sects. 3.1 and 3.2. Two distinct examples are given in Sects. 3.1.1 and 3.1.2. Section 4 discusses the optimal control of delayed systems and the advantages of DAE formulations of them. Finally some conclusions are in Sect. 5. Enough citations are given to enable the reader to follow up on a given comment, example, or application, but citing all relevant work would make the bibliography as long as the text and so many relevant citations are omitted.

2 Observer Design

Observers play a fundamental role in control theory and applications. There is an extensive literature on observers. Observers and the system being observed can be continuous or discrete time, deterministic or stochastic. We focus here on the continuous time deterministic case. The basic idea is that there is a dynamical system

$$F(\dot{x}, x, u, \psi, t) = 0, \tag{2.1a}$$

and an output equation

$$y = h(x, u, \psi, t). \tag{2.1b}$$

Here x is the state, u is the control or input, y is the output or measurements. ψ if present represents noise or uncertainty or faults. The particular assumptions on ψ will depend on which application is being discussed, but in general it is at least piecewise smooth. Both u and y are considered known. Unless necessary for clarity we delete the "(t)" from functions such as $x, y, u, \psi, \hat{x}, \hat{y}, z$. The goal is to get estimates $\hat{x}$ of x. Later we include ψ in Sect. 2.2.

An observer is another dynamical system for $\hat{x}$,

$$\hat{F}(\dot{\hat{x}}, \hat{x}, \hat{y}, y, u, t) = 0 \tag{2.2a}$$

and an output equation

$$\hat{y} = \hat{h}(\hat{x}, u, t). \tag{2.2b}$$

If $x(0) = \hat{x}(0)$ and $\psi = 0$, then $x = \hat{x}$ for all $t > 0$. If $\psi = 0$ and $x(0) - \hat{x}(0) \neq 0$, then we want $x - \hat{x} \to 0$ as $t \to \infty$. This convergence of $x - \hat{x}$ can be global or local if $x(0) - \hat{x}(0)$ has to be small to begin with where small is determined by the amount of nonlinearity. If $\psi \neq 0$, then we either want $x(0) - \hat{x}(0)$ to go to zero if ψ goes to zero fast enough or for $x(0) - \hat{x}(0)$ to become small if ψ is small.

System (2.1) is the actual physical system with variables to be estimated. All that is known is u and y. On the other hand (2.2) exists in software or hardware and $\hat{x}, u, \hat{y}, y, t$ are all known and available.

For the linear time invariant system

$$\dot{x} = Ax + Bu + R\psi \tag{2.3a}$$

$$y = Cx + Du + S\psi, \tag{2.3b}$$

the Luenberger observer takes the form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \tag{2.4a}$$

$$\hat{y} = C\hat{x} + Du \tag{2.4b}$$

and the error equation for the estimation error $e = x - \hat{x}$ is

$$\dot{e} = (A - LC)e + R\psi - LS\psi. \tag{2.5}$$

L is chosen to make $A - LC$ asymptotically stable if possible.

If A is $n \times n$, then

$$\Theta = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

is called the observability matrix. The pair $\{A, C\}$ is observable if $rank(\Theta) = n$. If $\{A, C\}$ is not observable, then the nullspace of Θ, $N(\Theta)$, is an A invariant subspace called the unobservable subspace. Eigenvalues of A restricted to $N(\Theta)$ are called the unobservable eigenvalues.

If $\{A, C\}$ is an observable pair, then L can be chosen to place the eigenvalues of $A - LC$ arbitrarily. $\{A, C\}$ is called detectable if any unobservable eigenvalues have negative real part. If $\{A, C\}$ is detectable, then L can be chosen so that $A - LC$ is asymptotically stable and the observable eigenvalues can be placed arbitrarily.

Observers for DAEs and observers which are DAEs have been discussed extensively in the literature. Notes [17, 24, 30, 35–40, 102] and the bibliography of [24]. Our emphasis here is different. We focus on how using a DAE, or a higher index DAE, or a higher dimensional DAE, can provide advantages over a more standard observer.

2.1 Nonlinear Observers

If the dynamics of the system being observed is nonlinear, then either the observer dynamics, or the error equation, or both are nonlinear. This makes the design of the observer so that the error equation is asymptotically stable more difficult. There are several approaches to trying to design the observer. One is to try to reformulate the problem so that the error equation becomes linear. If the error equation becomes linear, then it is much easier to stabilize the error equation by choosing L appropriately. This is the approach of this section.

Observers are usually formulated as explicit systems of differential equations and implemented using standard ODE solvers. In this section, we show that there can be advantages in formulating the observer as a DAE even if the system is originally an ODE. We first review the general idea of DAE observer design of Nikoukhah [78]. We then give two special normal forms for which DAE observer design yields an observer with linear error dynamics. The idea to use DAE observer normal forms is introduced on index one DAE observers and then extended to index two Hessenberg DAEs. This allows us to enlarge the class of nonlinear systems for which linear observer error dynamics can be achieved. This section is based on the work of Von Wissel [96] and von Wissel et al. [97]. Note also [56].

Consider the nonlinear systems

$$\dot{x} = f(x, u) \tag{2.6a}$$

$$y = h(x). \tag{2.6b}$$

where f and h are smooth vector fields on $\mathbb{R}^n$ and $\mathbb{R}^p$, and the p measurements y in (2.6b) are independent. The problem of observer design consists in finding a nonlinear system

$$0 = \hat{f}(\dot{\omega}, \omega, u, y) \tag{2.7a}$$

$$\hat{x} = \hat{g}(\omega, u, y) \tag{2.7b}$$

that generates an estimate $\hat{x}(t)$ of the true value $x(t)$.

There are essentially three approaches that have been used in the past for nonlinear observer design with a number of variations on each approach. The first approach is a natural extension of linear observers and is very commonly adopted, for example see the techniques presented in the comparative study of [100]. The other approach to observer design is to work directly with system equations (2.6), either formulating the estimation problem as a nonlinear algebraic system of equations which must be solved periodically using for example Newton's method, see for example [76], or formulating it as an optimization problem over some sliding finite horizon which is again solved periodically [75]. The third approach is to use specially designed Lyapunov equations to stabilize the dynamics of the nonlinear error equation.

We present an alternative to these three approaches. We show that there can be advantages in formulating the observer as a DAE which can then be solved using a numerical DAE solver. For index one DAEs the numerical integration can, for example, be done by the DAE solver DASSL [22, 82]. More importantly, if (2.6) has a special form and verifies some algebraic conditions, we can easily construct a DAE index one observer that has linear time invariant observer error dynamics.

The class of nonlinear systems is even larger if we allow (2.7) to be an index two Hessenberg DAE [22]. Index two Hessenberg DAEs are of particular interest since this type of DAE can also be safely solved by differential algebraic system solvers, for instance DASSL with fixed stepsize [22] or Radau5 [47]. The usefulness of this approach will be shown on a simple example. For a more detailed analysis of DAE index two design and its application to mechanical type problems see [96].

2.1.1 Index One DAE Observer

System (2.6) is a DAE in x since u and y are supposed to be known. This DAE is over-determined with n unknowns and $n + p$ equations. This DAE describes all the constraints that we have for constructing $\hat{x}$. To make this DAE numerically integrable, we can do relaxation by introducing a p-dimensional vector function