



LECTURE NOTES IN COMPUTATIONAL
SCIENCE AND ENGINEERING

107

Bert Jüttler · Bernd Simeon *Editors*

Isogeometric Analysis and Applications 2014

Editorial Board

T. J. Barth

M. Griebel

D. E. Keyes

R. M. Nieminen

D. Roose

T. Schlick

 Springer

Lecture Notes in Computational Science and Engineering

107

Editors:

Timothy J. Barth

Michael Griebel

David E. Keyes

Risto M. Nieminen

Dirk Roose

Tamar Schlick

More information about this series at <http://www.springer.com/series/3527>

Bert Jüttler • Bernd Simeon
Editors

Isogeometric Analysis and Applications 2014

 Springer

Editors

Bert Jüttler
Institut für Angewandte Geometrie
Johannes Kepler Universität Linz
Linz, Austria

Bernd Simeon
Fachbereich Mathematik
Technische Universität Kaiserslautern
Kaiserslautern, Germany

ISSN 1439-7358 ISSN 2197-7100 (electronic)
Lecture Notes in Computational Science and Engineering
ISBN 978-3-319-23314-7 ISBN 978-3-319-23315-4 (eBook)
DOI 10.1007/978-3-319-23315-4

Library of Congress Control Number: 2015959791

Springer Cham Heidelberg New York Dordrecht London
© Springer International Publishing Switzerland 2015

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Cover figure: Isogeometric simulation using a trivariate multi-patch NURBS model (from Pauley, Nguyen, Mayer, Špoh, Weeger, Jüttler, pp. 1–21)

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media
(www.springer.com)

Foreword for the Proceedings Volume

The very comfortable conference venue in Annweiler am Trifels, Germany, facing the Trifels Castle, perched on the peak of the Sonnenberg, high above the Queich valley, provided a beautiful and inspirational setting for **IGAA 2014, Isogeometric Analysis and Applications**, April 7–10, 2014. Trifels Castle is where Richard the Lionheart, King of England, was imprisoned for three weeks in the year 1193. History was made then and it was made again at **IGAA 2014**, now one of several very important, focused conferences that have been held to assess the state-of-the-art in the emerging field of Isogeometric Analysis. This field began with a single paper published in the 2005 October 1 issue of the international journal *Computer Methods in Applied Mechanics and Engineering*, entitled “Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement.” Almost immediately, Isogeometric Analysis attracted considerable attention in the research community, and at the present time it is enjoying explosive growth, as measured by the number of papers published on the topic and the citations to them in the literature. Currently, it represents perhaps the hottest topic within both Computational Mechanics and Computer-Aided Geometrical Modeling, two historically separate disciplines that have joined forces and embraced the vision of Isogeometric Analysis, that is, to reconstitute Computational Mechanics within the framework provided by Computational Geometry, thereby simplifying the analysis model development process and integrating engineering design and analysis into a unified framework. As research activity in Isogeometric Analysis has ramped up, industrial and commercial interest has followed, and we are now seeing considerable growth in these sectors as well. The promise of Isogeometric Analysis, that is, to improve the overall efficiency of the engineering product development process, is the driving force.

An interesting aspect of Isogeometric Analysis is the breadth of activities within it. One can pursue it from a very mathematical perspective, or a very practical engineering perspective, or anywhere in between. There seems to be something in it for everyone! It is a field that suits my broad scientific and engineering interests. From a mathematician’s point of view, I may be an engineer. From an engineer’s point of view, I may even be a mathematician. So be it. I would claim I am a

little bit of both, but I can also claim to be a Mechanical Design Engineer, at least I was 50 years ago! That was my first job at Grumman Aerospace Corporation (now Northrop Grumman) after obtaining my undergraduate degree in Mechanical Engineering. I enjoyed it and learned a lot about engineering in the real world, but it was not long before I gravitated toward engineering analysis. Design and analysis are the cornerstones of Isogeometric Analysis, and I feel I have practical experience with both. **IGAA 2014**, although including a little bit of everything, was somewhat weighted toward the mathematical side of the spectrum, reflecting the interests of the majority of the participants. Nevertheless, my invited talk at the conference was meant to be an overview, and I entitled it “Isogeometric Analysis: Where we are and where we are going.” In it, I also tried to identify a few subtopics that I felt were enjoying increased attention, particularly ones that might have substantial impact on the field going forward. I think some of the thoughts presented therein are worth recalling in this Foreword to the book of selected papers emanating from the conference.

Perhaps a main theme that needs to be highlighted is the analysis suitability of design models. Certainly, T-splines and LR-splines are having significant impact due to their watertight and unstructured qualities. An additional theme is performing analysis directly on trimmed NURBS files, which heretofore has been considered unsuitable for analysis. This is important because trimmed NURBS are ubiquitous in contemporary engineering design and give no sign of going away, at least anytime soon. Hierarchically refined NURBS are now reaching maturity and are becoming a standard tool in the design-through-analysis arsenal.

Another theme that has begun to manifest itself is the surprising efficiency of Isogeometric Analysis in practical applications. This is facilitated by the significant gain in accuracy per degree of freedom compared with classical C^0 -continuous finite element methods and decreased computational effort engendered by reduced integration and collocation techniques. Collocation techniques, which utilize the strong, rather than weak, form of the governing equations, are perhaps the ultimate reduced integration method. This topic has never had much success in the context of C^0 -continuous finite element methods, but is undergoing a renaissance in Isogeometric Analysis due to the smoothness of spline basis functions. Efficient solution algorithms that take advantage of the structure of Isogeometric Analysis matrix equations have also contributed to the gains in efficiency, as well as adaptive refinement strategies driven by a posteriori error estimates.

Other areas in which exciting developments are taking place include “thin” shell theory, which has also benefitted from smooth basis functions *without* derivative degrees of freedom. This has given new life to the classical Kirchhoff-Love shell theory, which requires C^1 -continuous basis functions in the classical Galerkin weak formulation. These are readily available in Isogeometric Analysis, with the added benefit of the elimination of rotational degrees of freedom. Further developments have even generalized this approach to the Reissner-Mindlin “thick” shell theory and at the same time have eliminated shear locking entirely, independent of basis functions. These indeed are significant developments and are expected to have a profound effect on structural analysis in the coming years.

Smooth, higher-order accurate basis functions and precise geometrical modeling have also created a new beginning in boundary element analysis, whose potential has never before been attained due to inaccurate, faceted geometrical approximations and low-order, C^0 -continuous basis functions. Several complete design-through-analysis procedures have been described using the Isogeometric Analysis version of boundary integral formulations. These have included shape optimization directly in the analysis-suitable CAD file, an enormous advantage in streamlining the design-through-analysis process. Shape and topology optimization are likewise general areas of significant Isogeometric Analysis activity.

The breadth of practical applications of Isogeometric Analysis is staggering, and some of applications to complex nonlinear problems and fluid-structure interaction are truly impressive. All this is being buttressed by many important mathematical studies involving complementary functional- and spectral-analysis approaches.

The papers in this volume only represent a small sampling of the many excellent presentations at **IGAA 2014**, but give one a glimpse of the significant recent progress in Isogeometric Analysis, which is rapidly becoming a mainstream analysis technology and a new paradigm for geometric design.

Austin, USA
June 23, 2015

Thomas J.R. Hughes

Preface

This book contains a selection of articles that were presented at the second workshop on *Isogeometric Analysis and Applications – IGAA 2014*, held in Annweiler am Trifels, Germany, from April 7 to 10, 2014. The emerging field of Isogeometric Analysis lives from the interaction of geometric modeling and numerical analysis, and the workshop was a great success in stimulating the exchange of ideas between these two fields. We are glad to see that, over the last years, this interaction has resulted in a series of groundbreaking scientific results, some of which are covered by the articles in this volume. The articles can be structured in four groups that cover the most important branches of recent developments in Isogeometric Analysis.

The first group of four papers deals with various aspects of multi-patch methods in Isogeometric Analysis.

In their paper entitled “Multipatch Discontinuous Galerkin Isogeometric Analysis,” U. Langer et al. present Galerkin-type methods that allow for discontinuities across the subdomain boundaries. The required interface conditions are weakly imposed by the terms associated with the boundary of the subdomains. The construction and the corresponding discretization error analysis of the resulting multi-patch schemes are given for heterogeneous diffusion model problems in 2D and 3D domains as well as on open and closed surfaces. The theoretical results are confirmed by extensive numerical experiments.

Next, E. Brivadis and her coauthors study the influence of quadrature errors on isogeometric mortar methods. After reviewing the recent mathematical theory, they investigate the effect of the errors caused by using numerical quadrature for the coupling integrals. Two approaches for designing quadrature rules and their impact on the convergence rate are analyzed.

M. Pauley et al. present a pipeline for the conversion of 3D models into a form suitable for Isogeometric Analysis. Given a boundary represented 3D model, they establish a chain of processes that result in a collection of volumetric NURBS patches. The suitability of the result for Isogeometric Analysis is also exemplified by performing stress simulations.

The fourth paper, entitled “Domain Decomposition Methods and Kirchhoff-Love Shell Multipatch Coupling in Isogeometric Analysis” and contributed by

A. Apostolatos et al., presents and compares several domain decomposition methods, including penalty and Lagrange multiplier methods. The methods, which are extended to account for geometrically nonlinear problems, provide highly accurate results, thus enabling the treatment of Kirchhoff-Love shells on multiple patches.

The second group of papers comprises three articles which are devoted to shell analysis and to T-splines. The last paper of the previous group also contributes to this topic.

C. Adam and coauthors propose a reduced shell element for Reissner-Mindlin geometric nonlinear analysis within the context of T-spline analysis. More precisely, the nonuniform Gauss-Legendre and patchwise reduced integrations for quadratic shape functions are both presented and compared to the standard full Gauss-Legendre scheme. The authors evaluate the effects of mesh distortion and local refinement, using both full and reduced numerical quadratures.

In their paper entitled “Multiresolution shape and topology optimisation with subdivision surfaces,” F. Cirak and K. Bandara review the multiresolution shape optimisation technique based on subdivision surfaces and present its application to elastic solids, electrostatic field equations, and thin shells. The geometry of the domain is described with subdivision surfaces, and different resolutions of the same surface are used for optimization and analysis. In their paper, the authors discretize elastic solids with the immersed finite element method, electrostatic field equations with the boundary element method, and thin shells with the subdivision finite element technique. In all situations there is no need to generate and to maintain an analysis-suitable volume discretization.

In the next paper on “Atom Simplification and Quality T-mesh Generation for Multi-resolution Biomolecular Surfaces” by T. Liao et al., the structure of biomolecules is first simplified using an error-bounded atom elimination method. An extended cross field-based parameterization method is then developed to adapt the parametric line spacings to different surface resolutions. From the parameterization results, adaptive and anisotropic T-meshes for the further T-spline surface construction are extracted. A gradient flow-based method is finally used to improve the T-mesh quality, with the anisotropy preserved in the quadrilateral elements.

The two papers in the next group address an important topic that is rarely discussed in detail in the literature, namely, algorithmic and implementation aspects in Isogeometric Analysis.

D. Fußeder and B. Simeon concentrate on these aspects in the context of isogeometric shape optimization. They present a self-contained treatment of gradient-based shape optimization with Isogeometric Analysis, focusing on issues such as the computation of shape gradients in an isogeometric formulation and how to update B-spline and NURBS geometries.

The other paper in this group, entitled “Effective Integration of Sophisticated Operators in Isogeometric Analysis with igatools,” has been contributed by N. Cavallini et al. It discusses aspects of igatools, which is a newly released library for operator assembly in Isogeometric Analysis. It is shown that the design of

the library makes it an effective tool in assembling isogeometric discretizations of sophisticated differential operators.

Finally, the two remaining papers address advanced topics from the theory of finite element methods and their application in Isogeometric Analysis.

S. Kleiss and S. Tomar present two-sided robust and sharp a posteriori error estimates in the isogeometric discretization of elliptic problems. These estimates, which are fully computable, provide robust, guaranteed, and sharp two-sided bounds of the exact error in the energy norm. The numerical realization and the quality of the computed error distribution are also covered.

The last paper of this volume, entitled “Multilevel Preconditioning for Variational Problems,” which has been contributed by A. Kunoth, surveys the main ingredients for multilevel preconditioners in terms of higher-order B-splines for standard second- and fourth-order elliptic partial differential equations in variational form possessing smooth solutions. It addresses three types of multilevel preconditioners for which asymptotic optimality can be shown. Two of them, the so-called additive preconditioners, are specified for Isogeometric Analysis involving linear elliptic partial differential operators in terms of variants of the BPX preconditioner and wavelet preconditioners. The third class is formed by multiplicative preconditioners, specifically by multigrid methods. The performance of the different preconditioners is demonstrated by numerical examples.

Last but not least, the editors acknowledge the support from many people and organizations that has been crucial in organizing the workshop and in compiling this volume. In particular, we are indebted to the reviewers. Their comments have helped us greatly to identify the manuscripts suitable for publication in this book and moreover were useful for improving many of the papers substantially. We also wish to express our thanks to Springer Verlag for the constructive and supportive cooperation during the production of this book. Financial support from the *Fraunhofer Institute for Industrial Mathematics (ITWM)*, Kaiserslautern, from the *Center for Mathematical and Computational Modeling (CM)²* at Technische Universität Kaiserslautern, and from the program for equal opportunities at Technische Universität Kaiserslautern made it possible to attract participants from all over the world to the workshop. We are now looking forward to the next event in the workshop series on *Isogeometric Analysis and Applications*.

Linz, Austria
Kaiserslautern, Germany
July 2015

Bert Jüttler
Bernd Simeon

Contents

Multipatch Discontinuous Galerkin Isogeometric Analysis	1
Ulrich Langer, Angelos Mantzaflaris, Stephen E. Moore, and Ioannis Touloupoulos	
The Influence of Quadrature Errors on Isogeometric Mortar Methods ...	33
Ericka Brivadis, Annalisa Buffa, Barbara Wohlmuth, and Linus Wunderlich	
The Isogeometric Segmentation Pipeline	51
Michael Pauley, Dang-Manh Nguyen, David Mayer, Jaka Špeh, Oliver Weeger, and Bert Jüttler	
Domain Decomposition Methods and Kirchhoff-Love Shell Multipatch Coupling in Isogeometric Analysis	73
Andreas Apostolatos, Michael Breitenberger, Roland Wüchner, and Kai-Uwe Bletzinger	
A Reduced Integration for Reissner-Mindlin Non-linear Shell Analysis Using T-Splines	103
C. Adam, S. Bouabdallah, M. Zarroug, and H. Maitournam	
Multiresolution Shape Optimisation with Subdivision Surfaces	127
Fehmi Cirak and Kosala Bandara	
Atom Simplification and Quality T-mesh Generation for Multi-resolution Biomolecular Surfaces	157
Tao Liao, Guoliang Xu, and Yongjie Jessica Zhang	
Algorithmic Aspects of Isogeometric Shape Optimization	183
Daniela Fußeder and Bernd Simeon	
Effective Integration of Sophisticated Operators in Isogeometric Analysis with <code>igatools</code>	209
Nicola Cavallini, Oliver Weeger, M. Sebastian Pauletti, Massimiliano Martinelli, and Pablo Antolín	

**Two-Sided Robust and Sharp a Posteriori Error Estimates
in Isogeometric Discretization of Elliptic Problems** 231
Stefan K. Kleiss and Satyendra K. Tomar

Multilevel Preconditioning for Variational Problems 247
Angela Kunoth