

Samson Abramsky  
Juha Kontinen  
Jouko Väänänen  
Heribert Vollmer  
Editors

# Dependence Logic

Theory and Applications



Birkhäuser





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Samson Abramsky  
Department of Computer Science  
University of Oxford  
Oxford, UK

Jouko Väänänen  
Department of Mathematics & Statistics  
University of Helsinki  
Helsinki, Finland

Juha Kontinen  
Department of Mathematics and Statistics  
University of Helsinki  
Helsinki, Finland

Heribert Vollmer  
Institut für Theoretische Informatik  
Leibniz Universität Hannover  
Hannover, Germany

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# Preface

In February 2013, the editors of this volume organized a Dagstuhl Seminar on “Dependence Logic: Theory and Applications” at the Schloss Dagstuhl Leibniz Center for Informatics in Wadern, Germany. This event consisted of both invited and contributed talks by some of the approximately 40 participants. After the conclusion of the seminar, the organizers invited a number of speakers to write surveys that present the state-of-the-art knowledge in their area of expertise. The Dagstuhl Seminar was followed by an Academy Colloquium of the Koninklijke Nederlandse Akademie van Wetenschappen (KNAW) and a further Dagstuhl Seminar “Logics for Dependence and Independence” in June 2015. Also from these latter meetings, a few participants were invited to contribute to this volume.

All contributions were peer-reviewed by experts in the field and revised before they were included in this volume.

We thank the Directorate of Schloss Dagstuhl and the board of the KNAW for their support, the speakers of the seminars for making all three meetings a successful event, the referees and, above all, the contributors of this volume for their informative and well-written articles. We also thank Benjamin Levitt at the publisher’s office for his support and guidance.

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Samson Abramsky  
Juha Kontinen  
Jouko Väänänen  
Heribert Vollmer



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# Contributors

**A. Blass** Mathematics Department, University of Michigan, Ann Arbor, MI, USA

**I. Ciardelli** ILLC, University of Amsterdam, Amsterdam, The Netherlands

**J. Corander** Department of Mathematics and Statistics, University of Helsinki, Helsinki, Finland

**A. Durand** IMJ-PRG, CNRS UMR 7586, Université Paris Diderot, Paris, France

**P. Galliani** School of Engineering and Informatics, University of Sussex, Falmer, Brighton, UK

**E. Grädel** Math. Foundations of Computer Science, RWTH Aachen University, Aachen, Germany

**Å. Hirvonen** Department of Mathematics and Statistics, University of Helsinki, Helsinki, Finland

**W. Hedges** Herons Brook, Sticklepath, Okehampton, Devon EX20 2PY, UK

**J. Kontinen** Department of Mathematics and Statistics, University of Helsinki, Helsinki, Finland

**S. Link** The University of Auckland, Auckland, New Zealand

**H. Nyman** Department of Mathematics and Statistics, Åbo Akademi University, Turku, Finland

**E. Pacuit** Department of Philosophy, University of Maryland, College Park, MD, USA

**J. Pensar** Department of Mathematics and Statistics, Åbo Akademi University, Turku, Finland

**J. Väänänen** Department of Mathematics and Statistics, University of Helsinki, Helsinki, Finland

Institute for Logic, Language and Computation, University of Amsterdam, Amsterdam, The Netherlands

**H. Vollmer** Institut für Theoretische Informatik, Leibniz Universität Hannover, Hannover, Germany

**F. Yang** Department of Philosophy and Religious Studies, Utrecht University, Utrecht, The Netherlands