BIRKHÄUSER

R.-D. Reiss
M. Thomas

Statistical Analysis of Extreme Values

with Applications to Insurance, Finance, Hydrology and Other Fields

Third Edition

Includes
CD-ROM

BIRKHÄUSER

R.-D. Reiss
M. Thomas

Statistical Analysis of Extreme Values

with Applications to Insurance, Finance, Hydrology and Other Fields

Third Edition

Birkhäuser
Basel · Boston · Berlin

Rolf-Dieter Reiss
Michael Thomas
FB Mathematik
Universität Siegen
Walter-Flex-Str. 3
57068 Siegen
Germany
e-mail: reiss@stat.math.uni-siegen.de
michael@stat.math.uni-siegen.de

2000 Mathematics Subject Classification 60G70, 62P05, 62P99, 90A09, 62N05, 68N15, 62-07, 62-09, 62F10, 62G07, 62G09, 62E25, 62G30, 62M10

Library of Congress Control Number: 2007924804

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the Internet at http://dnb.ddb.de

ISBN 978-3-7643-7230-9 Birkhäuser Verlag, Basel - Boston - Berlin

First edition 1997
Second edition 2001

Printed on acid-free paper produced from chlorine-free pulp. TCF ∞
Cover design: Alexander Faust, Basel, Switzerland
Printed in Germany

ISBN 978-3-7643-7230-9 e-ISBN 978-3-7643-7399-3

9 8 7 6 5 4 3 2 1 www.birkhauser.ch

Preface to the Third Edition

First of all we would like to thank all those who showed a steady, extraordinary interest in this book about extreme value analysis. This gave the motivation, courage and opportunity to provide an update of the book and to work out new topics, which primarily focus on dependencies, conditional analysis based on serial and covariate information, predictions, and the multivariate modeling of extremes.

As mentioned by Chris Heyde[1] while reviewing the second edition: "There is a considerable statistical content in the book, quite apart from its focus on extremes. ... The authors are seeking, quite properly, to embed the analysis of extreme values into the mainstream of applied statistical investigations." This strategy has been continued and strengthened in the new edition.

With each new edition there are complex questions as well as complex solutions. In that context, the cooperation with distinguished experts becomes more and more important.

Parts I–III about the basic extreme value methodology remained unchanged to some larger extent, yet notable are new or extended sections about

- *Testing Extreme Value Conditions with Applications* related to goodness–of–fit tests, co–authored by J. Hüsler and D. Li;
- *The Log–Pareto Model and other Pareto–Extensions* with a view towards super–heavy tailed distributions;
- *An Overview of Reduced–Bias Estimation*, an approach related to the jackknife method, co–authored by M.I. Gomes;
- *The Spectral Decomposition Methodology* which reduces multivariate questions to univariate ones to some extent,
- *About Tail Independence* with testing tail dependence against certain degrees of tail independence, co–authored by M. Frick.

[1] Heyde, C. (2002). Book Review: Statistical Analysis ... Australian & New Zealand J. Statist. 44, 247–248.

Of central importance are the new chapters entitled

- *Extreme Value Statistics of Dependent Random Variables* co–authored by H. Drees;
- *Conditional Extremal Analysis* which provides the necessary technical support for certain applications;
- *Elliptical and Related Distributions* with special emphasis on multivariate Student and sum–stable distributions.

Other new topics are collected within

Part IV: Topics in Hydrology and Environmental Sciences;

Part V: Topics in Finance and Insurance,

Part VI: Topics in Material and Life Sciences.

Within these parts one may find

- a new chapter about *Environmental Sciences*, co–authored by R.W. Katz, with a detailed description of the concepts of cycles, trends and covariates;
- a new section about *Predicting the Serial Conditional VaR*, co–authored by A. Kozek and C.S. Wehn, including remarks about the model validation;
- a new section about *Stereology of Extremes*, co–authored by E. Kaufmann, with remarks about modeling and estimation.

The entire text has been thoroughly updated and rearranged to meet the requirements. The new results and topics are elaborated on about 120 pages.

The book includes the statistical MS Windows application Academic Xtremes 4.1 and StatPascal on CD. The STABLE package for sum–stable distributions is no longer included. The major difference of the academic version compared to the professional one is a restriction of the executable sample sizes. Consequently, not all of the numerical examples in the book can be executed with the academic version.

To keep the book at a reasonable size, the former sections *Implementation in Xtremes* and the separate *Case Studies in Extreme Value Analysis* were omitted. The Appendix about Xtremes and StatPascal of the second edition is considerably shortened. The full description of Xtremes and StatPascal—also including the former sections *Implementation in Xtremes*—may be found in the *Xtremes User Manual* which is enclosed as a pdf–file on the CD.

It is a pleasure to thank Th. Hempfling who showed great efficiency in editing this book.

Siegen, Germany

Rolf–Dieter Reiss
Michael Thomas

Preface to the Second Edition

With the second edition we continue a project concerning extreme value analysis in combination with the interactive statistical software Xtremes which started in the late 1980's. An early publication was Chapter 6 together with the User's Guide to Xtremes in [16], besides tutorials for the statistical software Xtremes in 1993 and 1995 which had a wider circulation within the extreme value community. These efforts culminated in the first edition of the present book which has found a favorable reception from the side of practitioners and in academic circles.

The new highlights of this extended edition, elaborated on about 160 pages, include

- the statistical modeling of tails in conjunction with the global modeling of distributions with special emphasis laid on heavy–tailed distributions such as sum–stable and Student distributions;
- the Bayesian methodology with applications to regional flood frequency analysis and credibility estimation in reinsurance business;
- von Mises type upper bounds on remainder terms in the exceedance process approximation and a thorough theoretical and practical treatment of the phenomenon of penultimate distributions;
- a section about conditional extremes;
- an extension of the chapter about multivariate extreme value models, especially for the Gumbel–McFadden model with an application to the theory of economic choice behavior;
- a chapter about the bivariate peaks–over–threshold method;
- risk assessment of financial assets and portfolios in the presence of fat and heavy–tailed distributions by means of the Value–at–Risk (VaR);
- VaR under the Black–Scholes pricing and for general derivative contracts;
- sections about corrosion analysis and oldest–old questions.

The "analysis of extreme values must be embedded in various other approaches of main stream statistics" as mentioned in the first edition. In that context, M. Ivette Gomes[2] remarks "its scope is much broader, and I would rather consider it a welcome addition to the reference works in applied statistics ... though there is a unifying basis provided by extreme value theory." For the second edition, it is our declared aim to enforce this characteristic of providing a broad statistical background in the book.

In Part V there is a continuation of the successful program concerning self–contained "Case Studies in Extreme Value Analysis" of other authors. The case studies in the first edition are replaced by new ones with emphasis laid on environmental extreme value statistics. We thank Humberto Vaquera, José Villaseñor, Stuart Coles, Jürg Hüsler, Daniel Dietrich, Dietmar Pfeifer, Pieter van Gelder and Dan Lungu for the new contributions.

It was a pleasure to cooperate with several distinguished experts in various fields, namely, with John Nolan (sum–stable distributions), Edgar Kaufmann (rates of convergence and longevity of humans), Michael Falk (multivariate peaks–over–threshold), Jon Hosking (flood frequency analysis), Michael Radtke (insurance) and Casper de Vries and Silvia Caserta (finance).

The present statistical software environment is much more than an update of Xtremes, Version 2.1. As a consequence of our intention to establish a book about applied statistics—with a unifying basis provided by extreme value statistics—the Xtremes package becomes more and more applicable to various statistical fields. Further introductory remarks may be found in

- Xtremes: Overview and the Hierarchy
- Xtremes and StatPascal Within the Computing Environment RiskTec

after the Prefaces and at the beginning of the Appendix.

We would like to thank colleagues, readers and users of the first edition and Xtremes for their comments, questions and suggestions; among others, Claudio Baraldi, Arthur Böshans, Holger Drees, Harry Harper, Sylvia Haßmann, Claudia Klüppelberg, Elson Lee, Frank Marohn, Alexander McNeil, Richard Smith, Q.J. Wang, Carsten Wehn.

Siegen, Germany

Rolf–Dieter Reiss
Michael Thomas

[2]Gomes, M.I. (1999). Book Review: Statistical Analysis ... Extremes 2, 111–113.

Preface to the First Edition

This textbook deals with the statistical modeling and analysis of extremes. The restriction of the statistical analysis to this special field is justified by the fact that the extreme part of the sample can be of outstanding importance. It may exhibit a larger risk potential of random scenery such as floods, hurricanes, high concentration of air pollutants, extreme claim sizes, price shocks, incomes, life spans, etc. The fact that the likelihood of a future catastrophe is not negligible may initiate reactions which will help to prevent a greater disaster. Less spectacular yet important, the statistical insight gained from extremes can be decisive in daily business life or for the solution to ecological or technical problems.

Although extreme value analysis has its peculiarities, it cannot be looked at in an isolated manner. Therefore, the analysis of extreme values must be embedded in other various approaches of main stream statistics such as data analysis, nonparametric curve estimation, survival analysis, time series analysis, regression analysis, robust statistics and parametric inference.

The book is divided into

Part I: Modeling and Data Analysis;

Part II: Statistical Inference in Parametric Models;

Part III: Elements of Multivariate Analysis;

Part IV: Topics in Insurance, Finance and Hydrology;

Part V: Case Studies in Extreme Value Analysis,

Appendix: An Introduction to XTREMES.

Whenever problems involving extreme values arise, statisticians in many fields of modern science and in engineering or the insurance industry may profitably employ this textbook and the included software system XTREMES. This book is helpful for various teaching purposes at colleges and universities on the undergraduate and graduate levels. In larger parts of the book, it is merely presumed that the reader has some knowledge of basic statistics. Yet more and more statistical

prerequisites are needed in the course of reading this book. Several paragraphs and subsections about statistical concepts are intended to fill gaps or may be regarded as shorter refresher units. Parts I and II (with the exception of Chapter 6) are elementary yet basic for the statistical analysis of extreme values. It is likely that a more profound statistical background is helpful for a thorough understanding of the advanced topics in Chapter 6 and the multivariate analysis in Part III.

Part I sets out the statistical background required for the modeling of extreme values. The basic parametric models are introduced and theoretically justified in Chapter 1. The nonparametric tools introduced in Chapter 2 are most important for our approach to analyzing extreme values. In this context the included statistical software system is helpful to

- get a first insight into the data by means of visualizations;
- employ a data–based parametric modeling and assess the adequacy;
- draw statistical conclusions in a subjective manner;
- carry out the statistical inference in an objective (automatic) manner, and
- control results of parametric inference by nonparametric procedures.

Part II deals with statistical procedures in the parametric extreme value (EV) and generalized Pareto (GP) models. Yet, at the beginning, we start with the statistical inference in normal and Poisson models (Chapter 3) in order to give an outline of our approach to statistical questions within a setting which is familiar to a larger readership. From our viewpoint, the Gaussian model is merely relevant for the center of a distribution and, thus, not for extreme values. Chapters 4 and 5 develop the statistical methodology that is necessary for dealing with extremes. Applied questions are addressed in various examples. These examples also include critical examinations of case studies in publications which are occasionally very ambitious. We will approach extreme value analysis from a practical viewpoint, yet references are given to the theoretical background (as developed in the books [24], [20], [39], [42] and [16]). Applied contributions to extreme value analysis can be found in several journals. It is recommended to have a look at the J. Hydrology, Insurance: Mathematics and Economics, J. Econometrics, J. Royal Statistical Society B and, particularly, at the forthcoming journal Extremes, Statistical Theory and Applications in Science, Engineering and Economics. Other valuable sources for applied work are the recent Gaithersburg proceeding volumes [15] and the hydrology proceedings [13].

Part III contains supplementary material about the analysis of multivariate data and auxiliary results for multivariate distributions applied in Part II. Initially, a textbook for univariate extremes was scheduled. Yet, it is evident that a time–scale must be included in conjunction with time series phenomena, for example, exceedance times and exceedances are jointly visualized in a scatterplot. This was

our first step towards multivariate data. Further extensions of our initial framework followed so that, finally, we decided to include some procedures concerning multivariate extremes.

We also want to learn in which manner the methodology provided by the previous parts can be made applicable in certain areas. Part IV deals with important questions in

- insurance (coauthored by Michael Radtke),
- finance (coauthored by Casper G. de Vries),
- hydrology.

We hope that the explanations are also of interest for non–specialists.

Part IV has a certain continuation in Part V which contains several case studies in extreme value analysis. The case studies are written in the form of self–contained articles, which facilitated the inclusion of studies of other authors. One basic requirement for any case study is that the underlying data must be publicly accessible because, otherwise, the hypotheses and conclusions of an analyst cannot be critically examined and further improved by others, which would strongly violate scientific principles. We would like to thank Ana M. Ferreira, Edgar Kaufmann, Cornelia Hillgärtner, Tailen Hsing and Jürg Hüsler for their contributions.

The appendix is a manual for the included statistical software XTREMES. The menu–driven part of XTREMES allows us to surf through data sets and statistical models and reduces the "start up" costs of working in this area. For special problems one may employ the integrated programming language XPL. A short overview of the hierarchy of XTREMES is given after this preface. We believe that an experienced reader can partially handle XTREMES after having read the overview. A further link between the book and the statistical software is provided by sections entitled "Implementation in XTREMES" at the end of the chapters.

We will not make any attempt to give exhaustive references to the extreme value literature. Several footnotes provide hints to papers and books which are important from the historical viewpoint or may be helpful to get a more thorough understanding of special questions. The bibliography merely consists of references to monographs and proceeding volumes that are suggested for further reading or cited several times within the text.

We are grateful to several colleagues for valuable suggestions and stimulating discussions, especially, Sandor Csörgő, Richard A. Davis, Paul Embrechts, Michael Falk, Laurens de Haan, Jürg Hüsler, Edgar Kaufmann, Alex Koning, Ross Leadbetter, Wolfgang Merzenich, Wolfgang Wefelmeyer. We would like to express particular gratitude to the coauthors of Chapters 9 and 10 who had a constructive impact on our work. Very special thanks are due to Sylvia Haßmann for collaboration on the MS–DOS version of XTREMES (documented in [16]) and for assistance in writing a first draft of Section 6.1.

Several generations of students were exposed to the development of this book and the included software system; we acknowledge warmly the assistance of Simon Budig (final version of the UserFormula facility), Andreas Heimel (help system), Jens Olejak (minimum distance estimators), Claudia Schmidt (expert for the Moselle data) and Karsten Tambor (previous version of the multivariate mode of XTREMES). The technical assistance of Sarah Schultz and Maximilian Reiss was very helpful.

Part of the work of the first author was done as a guest professor at the Tinbergen Institute, Rotterdam, and visiting the Center for Stochastic Processes, Chapel Hill. Thanks are due to Laurens de Haan and Ross Leadbetter for their hospitality. The stimulating atmospheres of these institutions had a greater impact on the course of this work. The stay at the Tinbergen Institute also enabled the cooperation with Casper de Vries.

Siegen, Germany

Rolf–Dieter Reiss
Michael Thomas

Contents