#### Springer Texts in Business and Economics

H.A. Eiselt Carl-Louis Sandblom

## Operations Research

A Model-Based Approach

2nd Edition





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# **Operations Research**

A Model-Based Approach

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"And there is so much more value in learning why a set of conditions exists than simply accepting those conditions and committing them to memory."

Ben Stein

### **Preface**

Since the 1960s, operations research (or, alternatively, management science) has become an indispensable tool in scientific management. In simple words, its goal on the strategic and tactical levels is to aid in decision making and, on the operational level, automate decision making. Its tools are algorithms, procedures that create and improve solutions to a point at which optimal or, at least, satisfactory solutions have been found.

While many texts on the subject emphasize methods, the focus of this book is on the applications of operations research in practice. Typically, a topic is introduced by means of a description of its applications, a model is formulated, and its solution is presented. Then the solution is discussed and its implications for decision making are outlined. We have attempted to maximize the understanding of the topics by using intuitive reasoning while keeping mathematical notation and the description of techniques to a minimum. The exercises are designed to fully explore the material covered in the chapters, without resorting to mind-numbing repetitions and trivialization.

The book is designed for (typically second year) students of business management and industrial engineering. With the appropriate deletions, the material can be used for a one-semester course in the subject, while the complete material will be sufficient for a full-year course. The reasoning and explanations are intuitive throughout. Each algorithm is followed by a numerical example that shows in detail how the method progresses. After presenting the applications and the techniques, each chapter ends with a number of fully solved examples that review the concepts covered in the chapter. Some more technical material has been taken out and is available at <a href="http://esor.ie.dal.ca/">http://esor.ie.dal.ca/</a>. The second edition adds new material on multicriteria optimization, postman problems, Lagrangian relaxation, cutting planes, machine scheduling, and Markov chains.

It is our pleasure to thank all the people who have made this volume possible. Special thanks are due to Mr. Rauscher of Springer-Verlag for his encouragement and support in our writing of the second edition of this book, as well as Sina Raeisi

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H.A. Eiselt C.-L. Sandblom

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# **Symbols**

The list below includes the special symbols used in the book. While we have made every possible attempt to keep notation to a minimum, some symbolism will be required.

## **Notation**

- ∈: Element of
- $\subseteq$ : Subset
- ∪: Union of sets
- ∩: Intersection of sets
- Ø: Empty set
- |S|: Cardinality of the set S
- $x \in [a,b] : a \le x \le b$
- $x \in [a, b[: a \le x < b]$
- $x \in ]a,b] : a < x \le b$
- $x \in [a, b] : a < x < b$
- $\lceil x \rceil$ : Ceiling of x, the smallest integer greater or equal to x
- |x|: Floor of x, the largest integer smaller or equal to x
- |x|: Absolute value of x
- a = a + b: Valuation, a is replaced by a + b