2

Elements of Measurement Using the Slitting Method

2.1 Linear Elasticity and Superposition Principle

All mechanical methods of residual stress measurement are based on the principles of elasticity and linear superposition. In particular, the superposition for the slitting method as shown in Fig. 2.1, is extended from Bueckner's principle for crack propagation [9]. When a cut of depth a is introduced to a part with residual stress (case A), the stress on the site of cut is released (case B). This process is the same as imposing a stress field of the same magnitude of the stress in (case A) with a different sign on the site of the cut (case C), which leads to a stress-free slit face in case B. To compute the deformation or the compliance functions due to introduction of the cut in case B, we make use of case C because there is no change in deformation in case A. Note the superposition shown in Fig. 2.1 remains valid when external loads are present. For a body with prescribed displacement boundary conditions, however, the boundary condition should be properly maintained, as shown in Fig. 2.2. Note that the displacement at the boundary for case C should be set to zero. The stress estimated from the deformation measured from case B and the compliance functions computed from case C is due to both the residual stress and the prescribed boundary condition in case A.

There are situations where the condition of linear elastic deformation may be violated and the superposition is no longer applicable. For example, when a cut of size a is introduced to a thick ring with residual stress $\sigma_o(r)$ from the outer surface shown in Fig. 2.3, the change in stress $\Delta\sigma(r,a)$ near the inner surface 180° from the cut may be considerable and exceed the elastic limit if the increase has the same sign as the original residual stress [45]. In this case, the superposition shown in Fig. 2.1 will no longer be valid because of the presence of the plastic deformation. The superposition principle also assumes that the faces of cut are not in contact during cutting, which may take place if the cut is made into a zone of high compressive stress.

The use of superposition allows residual stress measurements to be carried out on several adjacent planes when the change of stress due to previous cuts

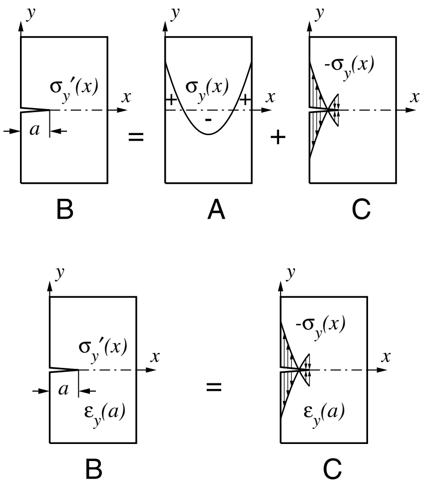

Fig. 2.1. Linear superposition for the slitting method.

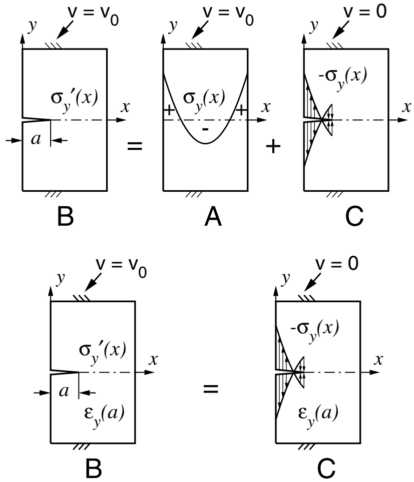

Fig. 2.2. Linear superposition with prescribed displacement conditions for the slitting method.

are included in the final stress estimation. Consider a welded plate, shown in Fig. 2.4, with a residual stress that varies with distance in both thickness and length. To obtain residual normal stress distributions on three planes in the weld and adjacent region, a cut is first made on plane I to measure residual stresses σ_I and τ_I. The release of σ_I and τ_I on plane I changes stresses on planes II and III from σ_{II} and σ_{III} to σ'_{II} and σ'_{III}, which can be measured by using the slitting method on each plane. From superposition, the original

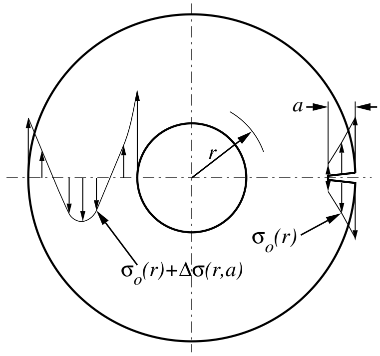

Fig. 2.3. The change of the stress near the inner surface may exceed the elastic limit for a thick-walled ring.

residual stresses on planes II and III are obtained as

$$\begin{aligned}\sigma_{II} &= \sigma'_{II} + \sigma^{I\sigma}_{II} + \sigma^{I\tau}_{II} \\ \sigma_{III} &= \sigma'_{III} + \sigma^{I\sigma}_{III} + \sigma^{I\tau}_{III}\end{aligned} \tag{2.1}$$

where $\sigma^{I\sigma}_{II}$, $\sigma^{I\tau}_{II}$, $\sigma^{I\sigma}_{III}$ and $\sigma^{I\tau}_{III}$ are the stresses computed on planes II and III by applying σ_I and τ_I respectively on the surfaces exposed by cut I. It is important to include residual shear stress τ_I in the computation if the stress field is not symmetric about plane I.

According to Saint-Venant's principle, the influence of releasing a residual stress should be mostly confined in a region of a dimension proportional to the size of the cut. Thus, for through-thickness measurement the change of the stress due to cutting is expected to become very small at a distance about one thickness from the plane of cut. This is confirmed by an analysis presented in Chapter 4 for a beam with an edge-crack.

2.2 Expressions for Approximation of Residual Stresses

Different measurements require different expressions for residual stresses. For a simply-connected 2-D body, shown in Fig. 2.5, an expression for residual normal stresses σ that varies through thickness must satisfy the following conditions

$$\int_0^1 \sigma(x)dx = 0$$

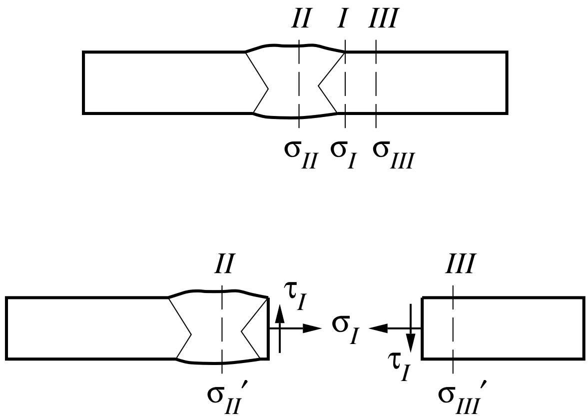

Fig. 2.4. Measurement of residual stresses on several planes in the weld region using the linear superposition.

$$\int_0^1 \sigma(x)(2x-1)dx = 0 \tag{2.2}$$

and for residual shear stresses τ,

$$\int_0^1 \tau(x)dx = 0$$
$$\tau(0) = \tau(1) = 0 \tag{2.3}$$

where, for simplicity, the distance x is normalized by the thickness.

It is well known that Legendre polynomials $L_i(x)$ of order $i \geq 2$ always satisfy Eq. (2.2). This can be easily verified by considering the orthogonality which states [70]

$$\int_0^1 L_i(x)L_j(x)dx = \frac{\delta_{ij}}{2i+1} \tag{2.4}$$

where $\delta_{ij} = 0$ if $i \neq j$. Since $L_0(x) = 1$ and $L_1(x) = 2x - 1$, Eq. (2.2) is guaranteed to hold when $\sigma(x)$ is replaced with $L_i(x)$ with $i \geq 2$. A continuous residual normal stress is thus always expressible by a Legendre polynomial series over the thickness as

$$\sigma(x) = \sum_{i=2}^{n} A_i L_i(x) \tag{2.5}$$

where A_i is the amplitude coefficient for $L_i(x)$. In computing Eq. (2.5), the actual expression for a Legendre polynomial is rarely used because it quickly becomes very lengthy. Instead, the recurrence relation is commonly used, which leads to very fast and efficient evaluation of Eq. (2.5). As an example, a subroutine in C programming language is given in Appendix C. It is possible to construct other functions that also satisfy Eq. (2.2) and can be used to represent a continuous residual normal stress, see Section 9.2 for an example. For a multiply-connected 2-D body, such as a ring, one or both of the conditions in Eq. (2.2) may not be required for the residual hoop stress through the wall-thickness.

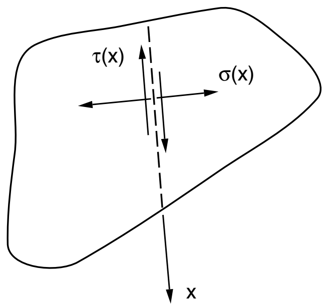

Fig. 2.5. Residual stresses on an arbitrary plane of a free body satisfy the equilibrium conditions.

An expression for residual shear stress, to the authors' knowledge, is not available in literature until recently probably due to much less attention received for the measurement of shear stresses. Its derivation is outlined here. First a general function that satisfies the second condition in Eq. (2.3) may be written as

$$\tau(x) = x(1-x)J(x) \tag{2.6}$$

which, when substituted into the first condition in Eq. (2.3), gives

$$\int_0^1 \tau(x)dx = \int_0^1 x(1-x)J(x)dx = 0 \tag{2.7}$$

This is a special case for the orthogonality of a class of Jacobi polynomials [70], which for the n^{th} order is given as

$$J_n(x) = \frac{(-1)^n}{n!x(1-x)}\frac{d^n}{dx^n}\{[x(1-x)]^{1+n}\} = \frac{Q_n(x)}{x(1-x)} \tag{2.8}$$

Thus, a general residual shear stress may be expressed as

$$\tau(x) = \sum_{i=1}^{n} B_i Q_i(x) \tag{2.9}$$

where B_i is the amplitude coefficient for the i^{th} order term. Again, the sum can be efficiently evaluated by using the recurrence relation, and a subroutine in C programming language is also included in Appendix C.

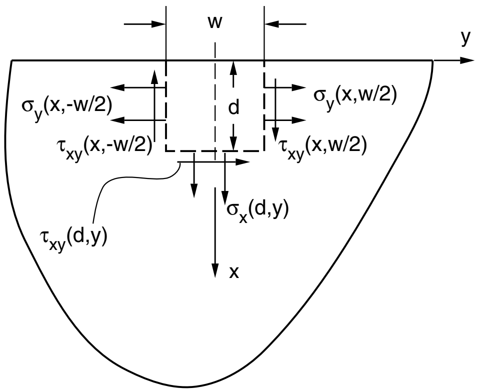

Fig. 2.6. A complete 2-D residual stress field on the site of a cut of finite width.

For near surface measurement no conditions on the resultant force and moment are required. However, the expression that describes the residual stresses released by a cut of finite width is more involved if the cut releases not only normal but other in-plane stresses.

For a slot of width w and depth d as shown in Fig. 2.6, the variation of the normal stress σ_y in a small region that contains the site of the cut may be sufficiently approximated by a second order function in y as below,

$$\sigma_y(x, y) = \sum_{j=0}^{2} y^j f_j(x) \tag{2.10}$$

where x-axis is chosen to coincide with the centerline of the cut. In practice variables x and y are often normalized by the final depth of cut for a near surface measurement or the thickness of part for through-thickness measurement. It is seen that $f_0(x)$ corresponds to the stress on the centerline of the slit. From equilibrium equations [124] for a 2-D stress field we find

$$\frac{\partial^2 \sigma_x}{\partial x^2} = \frac{\partial^2 \sigma_y}{\partial y^2} \tag{2.11}$$

which, when combined with Eq. (2.10), leads to

$$\frac{\partial^2 \sigma_x}{\partial x^2} = 2 f_2(x) \tag{2.12}$$

and

$$\sigma_x(x, y) = 2 \int_0^x dx \int_0^x f_2(x) dx + x A(y) + B(y) \tag{2.13}$$

Again, from equilibrium equations

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} &= -\frac{\partial \tau_{xy}}{\partial y} = 2 \int_0^x f_2(x) dx + A(y) \\ \frac{\partial \sigma_y}{\partial y} &= -\frac{\partial \tau_{xy}}{\partial x} = f_1(x) + 2y f_2(x) \end{aligned} \tag{2.14}$$

After integration we find

$$\begin{aligned} \tau_{xy}(x, y) &= -\int_0^x f_1(x) dx - 2y \int_0^x f_2(x) dx + C(y) \\ \tau_{xy}(x, y) &= -2y \int_0^x f_2(x) dx - \int A(y) dy + D(x) \end{aligned} \tag{2.15}$$

and

$$\begin{aligned} D(x) &= -\int_0^x f_1(x) dx \\ C(y) &= -\int A(y) dy \end{aligned}$$

The shear stress is thus given by

$$\tau_{xy}(x, y) = -2y \int_0^x f_2(x) dx - \int_0^x f_1(x) dx - \int A(y) dy \tag{2.16}$$

At the surface $x = 0$ we have

$$\tau_{xy}(0, y) = 0 \quad \textit{and} \quad \sigma_x(0, y) = 0 \tag{2.17}$$

Using Eq. (2.17) in Eqs. (2.13) and (2.16) yields

$$B(y) = 0 \quad \textit{and} \quad \int A(y) dy = 0 \tag{2.18}$$

Thus, we may set $A(y) = 0$. The final form of the stresses becomes

$$\begin{aligned} \sigma_x(x, y) &= 2 \int_0^x dx \int_0^x f_2(x) dx \\ \sigma_y(x, y) &= f_0(x) + y f_1(x) + y^2 f_2(x) \\ \tau_{xy}(x, y) &= -2y \int_0^x f_2(x) dx - \int_0^x f_1(x) dx \end{aligned} \tag{2.19}$$

Equation (2.19) shows the relationship among the three stress components in the region of the cut. For a very thin cut, $w/d \approx 0$, we have

$$\begin{aligned} \sigma_x(x,0) &= f_0(x) \\ \tau_{xy}(x,0) &= -\int_0^x f_1(x)dx \end{aligned} \tag{2.20}$$

where σ_x is omitted because it only acts on the bottom of cut. For a cut of finite width, the stresses to be released on the side faces of the cut ($y = \pm w/2$) are given by

$$\begin{aligned} \sigma_y(x,\pm\frac{w}{2}) &= f_0(x) \pm \frac{w}{2} f_1(x) + \frac{w^2}{4} f_2(x) \\ \tau_{xy}(x,\pm\frac{w}{2}) &= \mp w \int_0^x f_2(x)dx - \int_0^x f_1(x)dx \end{aligned} \tag{2.21}$$

and on the bottom of the cut of depth $x = d$ by

$$\begin{aligned} \sigma_x(d,y) &= 2\int_0^d dx \int_0^x f_2(x)dx \\ \tau_{xy}(d,y) &= -2y\int_0^d f_2(x)dx - \int_0^d f_1(x)dx \end{aligned} \tag{2.22}$$

Equation (2.21) shows that, as width w increases, the normal and shear stresses acting on one side of cut may become different from those on the other side. The expressions for σ_x and τ_{xy} on the bottom of cut in Eq. (2.22) represent a zero-order approximation and a linear approximation respectively. Note that stresses estimated using Eqs. (2.21) and (2.22) automatically satisfy the equilibrium conditions along the faces of the cut. Although Figure 2.3 shows a cut with a flat bottom, the solutions obtained above are equally valid for a cut with a curved bottom.

Equations (2.21) and (2.22) are also useful for through-thickness measurement when other stress components are not negligibly small. For a thin cut in particular, the zero-resultant force conditions over the thickness are satisfied when Eqs. (2.5) and (2.9) are combined with Eq. (2.20). That is,

$$\begin{aligned} \sigma_x(x,0) &= f_0(x) = \sum_{i=2}^{n} A_i L_i(x) \\ \tau_{xy}(x,0) &= -\int_0^x f_1(x)dx = \sum_{i=1}^{n} B_i Q_i(x) \end{aligned} \tag{2.23}$$

the second of which leads to

$$f_1(x) = -\sum_{i=1}^{n} B_i \frac{dQ_i(x)}{dx} \tag{2.24}$$

So far we have limited our discussion solely to 2-D stresses. For three-dimensional (3-D) stresses, a general expression for residual stresses that satisfies all the equilibrium conditions over a cross-section of an arbitrary shape is not yet available. Fortunately, the initial strain approach to be introduced in Chapter 9 provides a useful alternative to the conventional stress-based approach and allows a rigorous description of the residual stresses in parts of complex geometries.

After we have constructed an expression to approximate the residual stresses to be measured, the strain response to each function defined in the expression with a unit magnitude can be computed using one of the approaches to be presented in Chapters 3, 4 and 5. The strain obtained as a function of the depth of cut is referred to as the compliance function, and, therefore, we sometimes refer to the slitting method as the compliance method.

2.3 Experimental Procedures

Deformation due to releasing residual stress by a cut of progressively increasing depth can be measured as the change of displacements and/or strain. The latter one is by far the most commonly measured variable thanks to the wide availability of high precision electric-resistance strain gages of various sizes and patterns [41]. Although measurement of strain using strain gages offers a higher sensitivity and reliability than most displacement based measurements, it has certain limitations:

1. Measurement is limited to a few fixed locations;
2. As the number strain gages increases, the soldering and cabling of the gages becomes tedious and time-consuming;
3. Sensitive to temperature change if the gage's thermal expansion coefficient does not match that of the specimen adequately.

Fortunately, the slitting method in most cases requires measurement of strain only at one or two locations, as shown in Fig. 2.7A. When choosing a strain gage, it is crucial to match the thermal expansion coefficient of the strain gage with that of the surface on which the gage will be installed. Also, the gage length needs to be short enough to reduce the influence of strain gradient and increase the sensitivity of the measurement.

The method used to make a cut of increasing depth has evolved from sawing [71, 72], milling [11, 15, 16] to electric discharge machining (EDM) [38]. Sawing and milling are universally available but the cutting may introduce unwanted temperature increase and plastic deformation near the bottom of the cut. To reduce the effect of clamping force on the measurement, the plane of cut should be located sufficiently away from the fixture. A precise measurement of the depth of cut is more difficult when sawing is used. Furthermore, the release of a compressive residual stress may produce significant "binding" on the cutter to cause breakage, which often terminates a measurement prematurely. Electric discharge machining makes a cut without applying any forces, which minimizes the clamping force required to secure the specimen. For wire EDM, the location and depth of cut can be controlled precisely and cutting can be resumed in most cases after rethreading the wire if it breaks. For near surface measurement on a curved surface, conventional EDM is uniquely qualified to make a cut of nearly uniform depth, as shown in Fig. 2.7B, using a sheet of electrode that has a profile matching the curved surface. Because the electrode wears out gradually during cutting, the measurement of cut depth needs to be calibrated carefully for a given material and a given set of cutting conditions. The use of EDM has two main limitations:

1. It only cuts electrical conductive materials;
2. It is not portal for field applications.

There are two situations that require special attention during measurements. When a thin cut is made by a wire through a region of high compressive stress, the deformation due to releasing the stress may be so large that the faces of cut become in contact, which invalidates the assumption of the superposition principle. This situation can be corrected easily by cutting backward to remove the material in contact. On the other hand, a thin cut in a region of high tensile stress may initiate crack propagation near the tip of the cut, which terminates the test prematurely. In spite of these limitations, EDM remains the best method for making a high precision cut of progressively increasing depth for electrical conductive materials. For other materials, however, a mechanical method of cutting has to be considered.

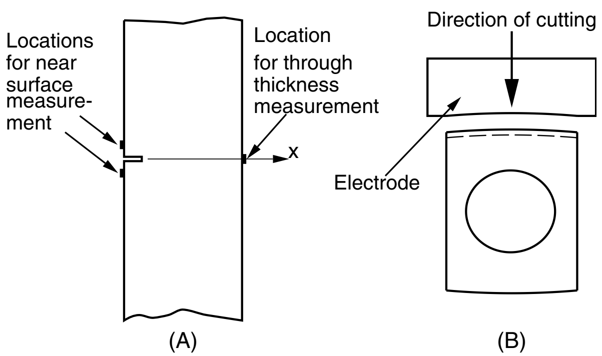

Fig. 2.7. (A) Locations for strain measurement. (B) Use of EDM to make a cut on a curved surface for near surface stress measurement.