

# CHAPTER 1

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## SETTING THE SCENE

**Abstract.** Although the importance of engaging students both cognitively and affectively when they learn mathematics is now widely recognized, the place of beliefs in the teaching and learning of mathematics is not well researched. After a brief introduction in which some contextual issues are raised, the contents of the contributions that follow – each with a clear focus on beliefs in mathematics education - are described in this introductory chapter.

### 1. INTRODUCTION

Mathematics is widely recognized not only as a core component of the curriculum but also as a critical filter to many educational and career opportunities. Yet in recent years much concern has been expressed about students' reluctance to continue with the study of mathematics well beyond the compulsory years, a trend often described emotionally as the drift away from mathematics and from the physical sciences. The passage below, taken from a national Australian document, is representative of the sentiments voiced.

Mathematics and science have a fundamental contribution to make both to understanding the world and to changing the world, particularly in the context of change and economic adjustment. The decline in interest in mathematics ... needs to be arrested. This is an urgent and complex matter related not only to education but to other issues. (Department of Employment, Education and Training, 1989, p. 14)

Lip service is certainly paid to the importance of engaging students of mathematics - of all ages - affectively as well as cognitively (see e.g., Jensen, Niss, & Wedege, 1998; the National Council of Teachers of Mathematics [NCTM], 2000). But (how) will that make a difference? What do we know about the interplay between beliefs and behaviors? Only a decade ago, McLeod (1992) noted: "(a)lthough affect is a central concern of students and teachers, research on affect in mathematics education continues to reside on the periphery of the field" (p. 575). Schoenfeld (1992) similarly argued that there was "a fairly extensive literature on" student beliefs, "a moderate but growing literature" about teacher beliefs, and as yet relatively little exploration of "general societal beliefs about doing mathematics" (p. 358). Research on mathematics and gender has been singled out as one area where "some aspects of beliefs about self have been researched quite thoroughly" (McLeod, 1992, p.580). A review of this work is beyond the scope of this

introductory chapter, but can be found in, for example, Fennema and Hart (1994), Forgasz and Leder (2001a, 2001b), Leder (1992), and Leder, Forgasz, and Solar (1996). At the same time, there are areas in research on beliefs, for example teacher change, where extensive research has been carried out over some two decades, but no consistent pattern has yet been identified for facilitating teacher change. Research reports about intervention programs commonly conclude that some teachers have changed, but others have not (e.g., Senger, 1999; Borko, Davinroy, Bliem, & Cumbo, 2000; Wood, 2001; Hart, this volume; Wilson & Cooney, this volume).

### *1.1. Genesis of the Book.*

Each of the contributions in this book has a clear focus on beliefs about mathematics. Aspects of beliefs about mathematics, its teaching and learning, are examined broadly and from a variety of perspectives. Collectively, the contributions reflect the diverse approaches used in the conceptualization and study of beliefs. The genesis of the book can be traced to a specialist international meeting about mathematics-related beliefs held in the unique setting of the *Mathematisches Forschungsinstitut Oberwolfach* in November 1999 (Pehkonen & Törner, 1999). The majority of the authors of the book were invited participants to the Institute, and their chapters are based on the presentations they gave at the meeting. Thus the core ideas contained in the chapters, and the diverse ways in which beliefs in mathematics education can be explored, were discussed beforehand, during an international high-level forum. The presentations and discussions were subsequently formalized, elaborated into full papers, and subjected to extensive peer-review.

## 2. ABOUT BELIEFS

A careful reading of the early psychological literature reveals that beliefs and belief systems began to be explored in the beginning of this century, particularly by social psychologists (Thompson, 1992). However, as the more behavioral perspectives of learning attracted increased attention, research endeavors turned to the more readily observed parts of human behavior, with a consequent loss of interest in beliefs. With new developments in cognitive science in the 1970s, attention to beliefs and belief systems re-emerged (Abelson, 1979). Although work on beliefs can be found in areas as diverse as political science, history, psychology, sociology, and anthropology, it is “especially social psychologists, who have devoted much effort into studying the acquisition and change of beliefs, their structure, their contents, and their effects mainly on individuals’ affect and behavior” (Bar-Tal, 1990).

Given the variety of perspectives and disciplines within which beliefs have been studied, it is not surprising that the field abounds with subtly different definitions and classifications of beliefs. A detailed overview of this work is again beyond the scope of this chapter, but can be found in later chapters in this volume (Furinghetti & Pehkonen; Op’t Eynde, de Corte, & Verschaffel). Here it is convenient to reproduce some key features highlighted by Bar-Tal (1990):

- The study of beliefs can be classified into four areas: “(a) acquisition and change of beliefs, (b) structure of beliefs, (c) effects of beliefs, and (d) content of beliefs<sup>1</sup>” (p. 12)
- “Beliefs have been viewed by social psychologists as units of cognition. They constitute the totality of an individual’s knowledge, including what people consider as facts, opinions, hypotheses, as well as faith” (p. 12). Descriptions such as this highlight the difficulty often shown in distinguishing between beliefs and knowledge<sup>2</sup>.
- “Beliefs [can] be differentiated on the basis in which they are formed: (a) Descriptive beliefs are formed on the basis of direct experience.... (b) Inferential beliefs ... are based on rules of logic that allow inferences.... (c) Informational beliefs are formed on the basis of information provided by outside sources...” (based on the work of Bem (1970) and Fishbein & Ajzen (1975), as summarized by Bar-Tal, p. 12).
- “Psychologists have suggested different features and dimensions to characterize beliefs.... Krech and Crutchfield (1948) proposed the following seven characteristics to describe beliefs: kind, content, precision, specificity, strength, importance, and verifiability” (p. 15).
- It is useful to focus on four characteristics of beliefs: “Confidence, centrality, interrelationship, and functionality. Confidence differentiates beliefs on the basis of truth attributed to them; centrality characterizes the extent of beliefs in individuals’ repertoire...; interrelationship indicates the extent to which the belief is related to other beliefs; functionality differentiates beliefs on the basis of the needs they fulfill” (p. 21).

Almost any one of these characterizations could have been used as an organizational theme for the various contributions in this volume. The grouping we finally selected is one we consider to be particularly constructive for the examination of beliefs about mathematics, its teaching and learning from a variety of different perspectives. We settled on a three clusters: contributions with a major focus on the concept of beliefs in mathematics education; on teachers’ beliefs; and on students’ beliefs.

### 3. ABOUT THE BOOK

As already indicated, the book is divided into three main sections, with different yet overlapping themes. The broad international mix of the contributing authors ensures a diversity of perspectives, as well as reference to relevant research beyond that published in English. Such coverage is less likely to be achieved with a culturally more homogeneous group of contributors. Thus the book offers a variety of different perspectives into the concept of beliefs, and into methods of investigating the place of beliefs in the teaching and learning of mathematics. A synthesis/critique chapter which, *inter alia*, highlights common and diverse themes, concludes each section.